



Comparing the Shape Parameters of Two Weibull Distributions Using Records: A Generalized Inference

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Abstract. The Weibull distribution is a very applicable model for the life-time data. For inference about two Weibull distributions using records, the shape parameters of the distributions are usually considered equal. However, there is not an appropriate method for comparing the shape parameters in the literature. Therefore, comparing the shape parameters of two Weibull distributions is very important. In this paper, we propose a method for constructing confidence interval and testing hypotheses about the ratio and difference of shape parameters using the concept of the generalized p -value and the generalized confidence interval. Simulation studies showed that our method is satisfactory. In the end, a real example is proposed to illustrate this method.

Keywords. Generalized p -value; generalized confidence interval; records; Weibull distribution.

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1 Introduction

Chandler (1952) introduced the concept of record value and studied some of its properties. Record data arise in a wide variety of practical situations; for example industrial stress testing, meteorological analysis, sporting and athletic events, and mining surveys. Properties of record data have been

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extensively studied in the literature. Ahsanullah (1995) and Arnold et al. (1998) are two good references about records and their properties.

Let X_1, X_2, \dots be a sequence of independent and identically distributed continuous random variables having the same distribution as the (population) random variable X . The random variable X_k is an upper record value if it is greater than all preceding values X_1, X_2, \dots, X_{k-1} . The sequence of record time $\{T_n, n \geq 0\}$ is defined as follows:

$$T_0 = 1 \quad \text{with probability } 1, \quad \text{and} \quad T_n = \min \{i : X_i > X_{T_{n-1}}\} \quad n \geq 1.$$

The sequence of upper record values is defined by $\{R_n = X_{T_n}, n = 0, 1, \dots\}$. By definition, X_1 is taken as the first upper record value. In the same way, an analogous definition can be provided for lower record values.

Suppose that we observe the first $n+1$ upper record values R_0, R_1, \dots, R_n from the cumulative distribution function (cdf) $F_{\boldsymbol{\theta}}(x)$ and the probability density function (pdf) $f_{\boldsymbol{\theta}}(x)$, where $\boldsymbol{\theta}$ is a vector of parameters. Then, the joint distribution of the first $n+1$ record values (for more details see Arnold et al., 1998, page 10) is given by

$$f_{\mathbf{R}}(\mathbf{r}) = f_{\boldsymbol{\theta}}(r_n) \prod_{i=0}^{n-1} \frac{f_{\boldsymbol{\theta}}(r_i)}{1 - F_{\boldsymbol{\theta}}(r_i)}, \quad r_0 < r_1 < \dots < r_n,$$

where $\mathbf{R} = (R_0, R_1, \dots, R_n)$ and $\mathbf{r} = (r_0, r_1, \dots, r_n)$.

Some researches have considered inference about the Weibull distribution based on records: Hoinkes and Padgett (1994) discussed the maximum likelihood estimates (MLE's) for both scale and shape parameters of a Weibull distribution. Chan (1998). and Sultan and Balakrishnan (1999) presented some inferential methods for the location-scale families of the distributions. Exact confidence intervals and exact joint confidence regions for the parameters of a Weibull distribution are derived by Chen (2004). Wu and Tseng (2006) proposed a computational approach for inference about the shape parameter. Soliman et al. (2006) obtained the MLE's for the parameters of a Weibull distribution and developed a Bayesian analysis using record values. Exact joint confidence regions for the parameters are also derived by Asgharzadeh and Abdi (2011), meanwhile Teimouri and Gupta (2012) proposed a confidence interval for n th upper/lower record value. Teimouri and Nadarajah (2013) derived exact expressions for constructing bias corrected MLE's. When the shape parameters of two Weibull distributions are equal, the stress-strength parameter of these distributions has a closed form.

For such a case, Baklizi (2012) proposed some methods for estimating and constructing confidence interval for the parameter of stress-strength reliability based on record values. However, it seems that there is no method for inference about the shape parameters of two Weibull distributions.

In this paper, we have considered constructing confidence interval and testing the hypothesis about the ratio (and difference) of two shape parameters. This is an extension of the method proposed by Wu and Tseng (2006) for the shape parameter of one Weibull distribution. For inference, we have applied the concepts of generalized confidence interval and generalized p -value introduced by Tsui and Weerahandi (1989) and Weerahandi (1993), respectively. These approaches have been used successfully to address several complex problems (see Weerahandi, 1995) such as inference about the mean of a Weibull distribution (Krishnamoorthy et al., 2009), the stress-strength reliability involving two independent Weibull distributions (Krishnamoorthy and Lin, 2010), the stress-strength reliability in two-parameter exponential distribution (Baklizi, 2013), inference on common mean of several normal populations (Krishnamoorthy and Lu, 2003), inference on common mean of several log-normal populations (Behboodian and Jafari, 2006) and comparing two generalized variances of multivariate distributions (Jafari, 2012).

The rest of the present article is organized as follows: In Section 2, we briefly review the concepts of generalized confidence interval and generalized p -value. A method for inference about the ratio and difference of two shape parameters is proposed in Section 3. In Section 4, we investigate the performance of the proposed approach using a simulation study. An illustrative example is proposed in Section 5.

2 Generalized p -value and Generalized Confidence Interval

Let \mathbf{X} be a random variable whose distribution depends on a vector parameters $\boldsymbol{\theta} = (\tau, \boldsymbol{\lambda})$, where τ is a scale parameter of interest and $\boldsymbol{\lambda}$ is a vector of nuisance parameters. Let \mathbf{x} denotes the observed value of \mathbf{X} . A generalized pivotal quantity for τ is a random quantity denoted by $T(\mathbf{X}; \mathbf{x}; \tau)$ and satisfies the following conditions:

- (i) The distribution of $T(\mathbf{X}; \mathbf{x}; \tau)$ is free of any unknown parameters.
- (ii) The value of $T(\mathbf{X}; \mathbf{x}; \tau)$ at $\mathbf{X} = \mathbf{x}$, i.e., $T(\mathbf{x}; \mathbf{x}; \tau)$ is free of the nuisance parameter $\boldsymbol{\lambda}$. In most cases, $T(\mathbf{x}; \mathbf{x}; \tau) = \tau$.

Appropriate percentiles of $T(\mathbf{X}; \mathbf{x}; \tau)$ form a confidence interval for τ . Specifically, if T_δ denotes the 100δ percentage point of $T(\mathbf{X}; \mathbf{x}; \tau)$, then $(T_{\gamma/2}, T_{1-\gamma/2})$ is a $100(1 - \gamma)\%$ generalized confidence interval for τ . Because, for a given \mathbf{x} , the distribution of $T(\mathbf{X}; \mathbf{x}; \tau)$ does not depend on any unknown parameters, its percentiles can be found.

In the above setup, suppose that we are interested in testing the hypotheses

$$H_0 : \tau \leq \tau_0 \quad vs. \quad H_1 : \tau > \tau_0, \quad (1)$$

for a specified known τ_0 . The generalized test variable, denoted by $T^*(\mathbf{X}; \mathbf{x}; \tau)$, is defined as follows:

- (i) The value of $T^*(\mathbf{X}; \mathbf{x}; \tau)$ at $\mathbf{X} = \mathbf{x}$ is free of any unknown parameters.
- (ii) The distribution of $T^*(\mathbf{X}; \mathbf{x}; \tau)$ is stochastically monotone (i.e., stochastically increasing or stochastically decreasing) in τ for any fixed \mathbf{x} and λ .
- (iii) The distribution of $T^*(\mathbf{X}; \mathbf{x}; \tau)$ is free of any unknown parameters.

Let $t^* = T^*(\mathbf{x}; \mathbf{x}; \tau_0)$ denotes the observed value of $T^*(\mathbf{X}; \mathbf{x}; \tau)$ at $(\mathbf{X}; \tau) = (\mathbf{x}; \tau_0)$. When the above three conditions in (i)-(iii) hold, the generalized p -value for testing the hypotheses in (1) is defined as

$$p = P(T^*(\mathbf{X}; \mathbf{x}; \tau_0) \leq t^*), \quad (2)$$

if $T^*(\mathbf{X}; \mathbf{x}; \tau)$ is stochastically decreasing in τ . In many situations, $T^*(\mathbf{X}; \mathbf{x}; \tau) = T(\mathbf{X}; \mathbf{x}; \tau) - \tau$, where $T(\mathbf{X}; \mathbf{x}; \tau)$ is a generalized pivotal variable. The test based on the generalized p -value rejects H_0 when the generalized p -value is smaller than a given level γ . However, the size and the power function of such a test may depend on the nuisance parameters.

For more details on generalized p -values and generalized confidence intervals, we refer readers to Weerahandi (1995).

3 Inference About the Parameters

The Weibull distribution with parameters α and β has the pdf

$$F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta}, \quad x > 0, \quad \alpha > 0, \quad \beta > 0,$$

and the cdf

$$f(x) = \frac{\beta}{\alpha^\beta} x^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^\beta}, \quad x > 0.$$

This distribution is a generalization of the exponential distribution and the Rayleigh distribution. Also, $Y = \log(X)$ has extreme value (Gumbel) distribution with parameters $b = \frac{1}{\beta}$ and $a = \log(\alpha)$, when X has a Weibull distribution with parameters α and β . It is a well-known distribution that is widely used for lifetime models while having numerous varieties of shapes and being very flexible such that it has both increasing and decreasing failure rates. Based on this, the Weibull distribution is used for many applications such as hydrology, reliability engineering, weather forecasting and insurance.

Suppose $\mathbf{R}_i = (R_{i0}, R_{i1}, \dots, R_{ini})$, $i = 1, 2$ are the set of records corresponding to an independent and identically sequence of a Weibull distribution with parameters α_i and β_i . In this section, we consider constructing confidence interval for the ratio of the shape parameters, $\pi = \frac{\beta_1}{\beta_2}$, and testing the one-sided hypotheses

$$H_0 : \pi \leq \pi_0 \quad vs. \quad H_1 : \pi > \pi_0, \quad (3)$$

and the two-sided hypotheses

$$H_0 : \pi = \pi_0 \quad vs. \quad H_1 : \pi \neq \pi_0, \quad (4)$$

where π_0 is a specified value.

The cdf of the record values, \mathbf{R}_i , can be written as

$$f_{\mathbf{R}_i}(\mathbf{r}_i) = \frac{\beta_i^{n_i+1}}{\alpha_i^{\beta_i(n_i+1)}} e^{-\left(\frac{r_{ini}}{\alpha_i}\right)^{\beta_i}} \prod_{j=0}^{n_i} r_{ij}^{\beta_i-1}, \quad 0 < r_{i0} < r_{i1} < \dots < r_{ini},$$

where $\mathbf{r}_i = (r_{i0}, r_{i1}, \dots, r_{ini})$. Therefore, $\left(R_{ini}, \prod_{j=0}^{n_i} R_{ij}\right)$ is a sufficient statistic for (α_i, β_i) . In addition, the MLE's of the parameters α_i and β_i are (see Soliman et al., 2006):

$$\hat{\beta}_i = \frac{n_i + 1}{\sum_{i=0}^n \log\left(\frac{R_{ini}}{R_{ij}}\right)}, \quad \hat{\alpha}_i = \frac{R_{ini}}{(n_i + 1)^{\frac{1}{\hat{\beta}_i}}}. \quad (5)$$

Based on the equality of shape parameters of two Weibull distributions, i.e. $\beta_1 = \beta_2 = \beta$, the joint density function of these record values can be

written as

$$f_{\mathbf{R}_1, \mathbf{R}_2}(\mathbf{r}_1, \mathbf{r}_2) = \frac{\beta^{n_1+n_2+2}}{\alpha_1^{\beta(n_1+1)} \alpha_2^{\beta(n_2+1)}} e^{-\left(\frac{r_{1n_1}}{\alpha_1}\right)^\beta - \left(\frac{r_{2n_2}}{\alpha_2}\right)^\beta} \prod_{j=0}^{n_1} r_{1j}^{\beta-1} \prod_{h=0}^{n_2} r_{2h}^{\beta-1}.$$

Therefore, $(R_{1n_1}, R_{2n_2}, \prod_{j=0}^{n_1} R_{1j} \prod_{h=0}^{n_2} R_{2h})$ is a sufficient statistic for $(\alpha_1, \alpha_2, \beta)$, and the MLE's of the parameters α_1, α_2 and β are (see Baklizi, 2012):

$$\hat{\beta} = \frac{n_1 + n_2 + 2}{\sum_{j=0}^{n_1} \log\left(\frac{R_{1n_1}}{R_{1j}}\right) + \sum_{j=0}^{n_2} \log\left(\frac{R_{2n_2}}{R_{2j}}\right)}, \quad \hat{\alpha}_i = \frac{R_{in_i}}{(n_i + 1)^{\frac{1}{\hat{\beta}}}} \quad i = 1, 2. \quad (6)$$

Wu and Tseng (2006) has proposed an approach for inference about the shape parameter of a Weibull distribution. We will use this method for inference about π , and propose a generalized confidence interval for this parameter as well as a generalized test variable for testing the hypotheses in (3) and (4).

Let

$$W_i(\beta_i) = \frac{\sum_{j=0}^{n_i} R_{ij}^{\beta_i}}{(n_i + 1) \left(\prod_{j=0}^{n_i} R_{ij}\right)^{\frac{\beta_i}{n_i+1}}}, \quad i = 1, 2.$$

Wu and Tseng (2006) show that $W_i(\beta_i)$ is an increasing function with respect to β_i . Also, the distribution of $W_i(\beta_i)$ does not depend on parameters α_i and β_i . In fact, $W_i(\beta_i)$ is distributed as

$$W_i^* = \frac{\sum_{j=0}^{n_i} R_{ij}^*}{(n_i + 1) \left(\prod_{j=0}^{n_i} R_{ij}^*\right)^{\frac{1}{n_i+1}}},$$

where $R_{i0}^*, R_{i1}^*, \dots, R_{in_i}^*$ is the record values from the standard exponential distribution. However, the exact distribution of $W_i(\beta_i)$ is very complicated, and its percentiles are obtained using the Monte Carlo simulation.

Let

$$\mathbf{g}_i(\beta_i) = \frac{\sum_{j=0}^{n_i} r_{ij}^{\beta_i}}{(n_i + 1) \left(\prod_{j=0}^{n_i} r_{ij}\right)^{\frac{\beta_i}{n_i+1}}} - \frac{\sum_{j=0}^{n_i} R_{ij}^*}{(n_i + 1) \left(\prod_{j=0}^{n_i} R_{ij}^*\right)^{\frac{1}{n_i+1}}}, \quad (7)$$

where r_{ij} is the observed value of R_{ij} , $i = 1, 2, j = 0, 1, \dots, n_i$, and $R_{i0}^*, R_{i1}^*, \dots, R_{in_i}^*$ are the record values from the standard exponential distribution.

Theorem 1. Let T_i be the solution of the following equations with respect to β_i :

$$g_i(\beta_i) = 0, \quad i = 1, 2. \tag{8}$$

Then

- i. T_i is unique.
- ii. T_i is a generalized pivotal variable for β_i .

Proof. i. Consider $\bar{R}_i^* = \frac{1}{n_i+1} \sum_{j=0}^{n_i} R_{ij}^*$ and $\bar{G}_i^* = \left(\prod_{j=0}^{n_i} R_{ij}^* \right)^{\frac{1}{n_i+1}}$, $i = 1, 2$ are the arithmetic mean and geometric mean of $R_{i0}^*, R_{i1}^*, \dots, R_{in_i}^*$. It is well-known $\bar{G}_i^* < \bar{R}_i^*$. Therefore,

$$\lim_{\beta_i \rightarrow 0} g_i(\beta_i) = 1 - \frac{\bar{G}_i^*}{\bar{R}_i^*} < 0, \quad \lim_{\beta_i \rightarrow \infty} g_i(\beta_i) = \infty.$$

Also, $g_i(\beta_i)$ is an increasing function with respect to β_i (for more details see Wu and Tseng, 2006). So, T_i is unique.

ii. It is obvious using the substitution approach described by Weerahandi (2004), page 24. □

Based on the Theorem 1, it can be understood that i) the observed value of T_i is β_i and does not depend on the nuisance parameter, α_i , and ii) the distribution of T_i does not depend on any parameter. Now define

$$G = \frac{T_1}{T_2}. \tag{9}$$

Therefore, G is a generalized pivotal variable for π and can be used for constructing confidence interval for this parameter. A generalized test variable can also be defined as

$$G^* = G - \pi.$$

The cdf of G^* is $F_{G^*}(x) = F_G(x + \pi)$, where $F_G(\cdot)$ is the cdf of the generalized pivotal variable G in (9) and does not depend on any parameter. Therefore, $F_{G^*}(x)$ is an increasing function with respect to π , and G^* is stochastically decreasing with respect to π , and the generalized p -values for testing the one-sided hypothesis in (3) and (4) are

$$p = P(G^* < 0 | \pi_0) = P(G < \pi_0), \tag{10}$$

$$p = 2 \min \{P(G < \pi_0), P(G > \pi_0)\}, \tag{11}$$

respectively. This generalized confidence interval and the generalized p -values can be obtained using Monte Carlo simulation. To do this, an algorithm is given in Section 4.

Remark 1. A generalized pivotal approach can also be defined for difference between two shape parameters, $\beta_1 - \beta_2$ as $H = T_1 - T_2$.

4 Simulation Study

A simulation study is performed to assess the accuracy of the proposed generalized procedure. We evaluated the coverage probability and the expected length of the 95% generalized confidence about $\pi = \beta_1/\beta_2$. To do this, without loss of generality, we set $\alpha_1 = \alpha_2 = 1$ and use Monte Carlo simulation by the following algorithm:

Algorithm 1. For given β_1 and β_2 ,

1. Two sets of records, r_{i0}, \dots, r_{in_i} , ($i = 1, 2$) were generated from the Weibull distributions.
2. Generate the record values $R_{i0}^*, \dots, R_{in_i}^*$ from the standard exponential distribution.
3. Write the equations $g_i(\beta_i)$, $i = 1, 2$ in (7) and obtain T_i by solving the equations in (8).
4. Calculate $G = T_1/T_2$.
5. Repeat Steps 2-4, $M = 10,000$ times and obtain the values G_1, \dots, G_M .
6. Sort the values of G_l , denoted by $G_{(1)}, \dots, G_{(M)}$. The $100(1 - \gamma)\%$ generalized confidence for π is $[G_{(\gamma M/2)}, G_{((1-\gamma/2)M)}]$.
7. Set $D_l = 1$ if $G_{(\gamma M/2)} < \frac{\beta_1}{\beta_2} < G_{((1-\gamma/2)M)}$, otherwise $D_l = 0$.
8. Repeat Steps 1-7, $N = 10000$ times. Then coverage probability is $\frac{1}{N} \sum_{l=1}^N D_l$.

For $\beta_2 = 2$, and some selected values for β_1 , n_1 , and n_2 , the coverage probabilities and the expected lengths of the generalized confidence interval, with 10000 repetition, are given in Table 1. Empirically, we can conclude that

- i. The coverage probability of our method is close to the nominal confidence coefficient.
- ii. For fixed n_1 and n_2 , the expected length of the method is increasing in the parameter shape, β_1 .
- iii. For fixed β_1 and for fixed n_1 , the expected length of the method is decreasing in n_2 .
- iv. For fixed β_1 and for fixed n_2 , the expected length of the method is decreasing in n_1 .

Table 1. Empirical coverage probabilities and expected lengths of the 95% generalized confidence interval.

		β_1						
		n_1, n_2	0.5	1.0	1.2	1.5	2.0	3.0
Empirical Coverage	3,3	0.946	0.952	0.953	0.946	0.948	0.949	0.952
	3,7	0.952	0.948	0.951	0.953	0.954	0.947	0.948
	3,14	0.950	0.952	0.951	0.948	0.953	0.946	0.944
	7,3	0.956	0.948	0.952	0.946	0.953	0.954	0.951
	7,7	0.953	0.951	0.948	0.950	0.953	0.947	0.945
	7,14	0.952	0.952	0.953	0.947	0.946	0.953	0.954
	14,3	0.945	0.952	0.954	0.950	0.947	0.949	0.952
	14,7	0.948	0.953	0.945	0.949	0.952	0.954	0.944
	14,14	0.951	0.948	0.950	0.950	0.949	0.946	0.953
Expected Length	3,3	2.567	4.372	5.440	6.306	9.438	11.664	24.562
	3,7	1.266	2.681	3.503	4.179	5.244	9.541	13.034
	3,14	0.987	2.463	2.221	3.443	3.851	6.891	11.660
	7,3	1.567	2.786	4.116	5.200	5.852	10.089	15.026
	7,7	0.908	1.680	2.157	2.705	3.390	5.169	8.050
	7,14	0.641	1.306	1.648	2.025	2.904	3.958	6.460
	14,3	1.418	2.608	3.522	4.425	5.310	7.814	13.786
	14,7	0.711	1.521	1.698	2.125	3.062	4.191	7.348
	14,14	0.523	1.077	1.207	1.691	2.040	2.913	5.195

5 An Illustrative Example

In this section, we have consider a real data, due to Nelson (1982), concerning the data on time to breakdown of an insulating fluid between electrodes at two voltages of 34 and 36 kV (minutes). This data set is also given by Lawless (2003, page 3). The times to breakdown at voltages of 34 kV and 36 kV are given bellow;

Voltage of 34 kV:	0.96	4.15	0.19	0.78	8.01	31.75	7.35	6.50
	8.27	33.91	32.52	3.16	4.85	2.78	4.67	1.31
	12.06	36.71	72.89					
Voltage of 36 kV:	1.97	0.59	2.58	1.69	2.71	25.50	0.35	0.99
	3.99	3.67	2.07	0.96	5.35	2.90	13.77	

Therefore, the upper record values at voltage of 34 kV are 0.96, 4.15, 8.01, 31.75, 33.91, 36.71, 72.89, and at voltage of 36 kV are 1.97, 2.58, 2.71, 25.50.

A model suggested by engineering considerations is that, for a fixed voltage level, the time to breakdown has a Weibull distribution (Soliman et al., 2006). Based on (5), the MLE's of the parameters are $\hat{\beta}_1 = 0.5990$, $\hat{\beta}_2 = 0.5639$, $\hat{\alpha}_1 = 2.8303$, $\hat{\alpha}_2 = 2.1822$, and their standard errors using the Hessian matrix are $s.e.(\hat{\beta}_1) = 0.2264$, $s.e.(\hat{\beta}_2) = 0.2820$, $s.e.(\hat{\alpha}_1) = 3.9072$, $s.e.(\hat{\alpha}_2) = 3.3074$.

The %95 generalized confidence interval for $\pi = \beta_1/\beta_2$ is (0.2550, 4.9537). At the same time, %95 generalized confidence interval for $\beta_1 - \beta_2$ is (-0.7849, 0.7283). Also, we consider testing the equality of shape parameters of two Weibull distributions, i.e. $H_0 : \beta_1 = \beta_2$ vs. $H_1 : \beta_1 \neq \beta_2$. Using the Algorithm 1 with $\pi_0 = 1$, the generalized p -value for testing this hypotheses is 0.9830. So, it can be concluded that the shape parameters of two Weibull distributions are equal, i.e. $\beta_1 = \beta_2 = \beta$ at level 0.05. In this case, the MLE's of all parameters are $\hat{\beta} = 0.5857$, $\hat{\alpha}_1 = 2.6297$, $\hat{\alpha}_2 = 2.3916$, and their standard errors are $s.e.(\hat{\beta}) = 0.1766$, $s.e.(\hat{\alpha}_1) = 3.1333$, $s.e.(\hat{\alpha}_2) = 2.6609$.

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