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Bayes Estimation for a Simple Step-stress Model with Type-I Censored Data from the Geometric Distribution

A. Arefi, M. Razmkhah* and G. R. Mohtashami Borzadaran

Ferdowsi University of Mashhad

Abstract. This paper focuses on a Bayes inference model for a simple step-stress life test using Type-I censored sample in a discrete set-up. Assuming the failure times at each stress level are geometrically distributed, the Bayes estimation problem of the parameters of interest is investigated in the both of point and interval approaches. To derive the Bayesian point estimators, some various balanced loss functions are used. Furthermore, a simulation study and sensitivity analysis is performed to carry out the performance of the results of the paper. An example is also presented to illustrate the proposed procedure. Finally, some conclusions are stated.

Keywords. Balanced loss function; Dirichlet prior distribution; highest posterior density (HPD) intervals; order statistics.

MSC 2010: 62N05, 62F15, 62N01.

1 Introduction

The accelerated life-testing experiments have been offered to obtain the life data in the situations of reliability analysis in which the experiment may not terminate on an adequate time under the normal conditions. Nelson (1990), Meeker and Escobar (1998) and Bagdonavicius and Nikulin (2002) are some key references in the area of accelerated life-testing experiments include the step-stress accelerated life testing (SSALT) in which the stress

* Corresponding author

levels of the experiment are increased step-by-step over time. In an SSALT, the products are tested under different levels of stresses, such as temperature, voltage, pressure and etc., or some combination of these factors. The SSALT with only two levels of stresses is referred to as a simple SSALT. In the situations in which the experimental units are placed at low and high levels of stresses, a simple SSALT is indeed performed. For example, suppose that some electronic products are simultaneously placed on a test under initial voltage 300V and the voltage increases to 400V after 30 minutes from the beginning of the test. The data gathered from this scheme must be analyzed with a simple SSALT. This model has been considered for the exponential distribution in references such as Balakrishnan and Xie (2007), Balakrishnan and Han (2008) and Balakrishnan et al. (2009). This problem has also been studied by some authors in a Bayesin point of view. For example, DeGroot and Goel (1979) proposed a Bayesian inference model and presented an optimality criterion for simple SSALT. Van Dorp et al. (1996) and Van Dorp and Mazzuchi (2004) developed a general Bayes inference model for accelerated life testing, assuming the failure times at each stress level are exponentially distributed.

Let us now consider the accelerated life-testing experiments in a discrete set-up such that the life times of the units on test depend on the number of shocks the units receive or the number of times they are switched on and off, for example. In this set-up, if w be the number of switch on and off or shocks the units receive until they fail, the integer value w is considered as the failure time of a unit on test. Suppose n identical units are simultaneously placed on a test under an initial stress level s_1 and the stress levels are increased to $s_2, \ldots, s_m$ at the pre-fixed times $w_1, \ldots, w_{m-1}$. For the statistical analysis of geometric distribution under the SSALT, see, Xu et al. (2010) and Wang et al. (2012). In the situations in which the experiment has to terminated before or at the pre-determined time w_m, the Type-I censoring scheme is indeed performed. Some researches have been done based on the censored samples in the discrete set-up such as Rezaei and Arghami (2002), Davarzani and Parsian (2011) and Balakrishnan et al. (2011). Arefi and Razmkhah (submitted) investigated the statistical analysis for a simple SSALT in a frequentist approach assuming the failure times at each level of stress follow a geometric distribution. The main goal of this paper is to study the problem of Bayes estimation of the parameters of interest in such a model. Ahmadi et al. (2009) investigated the Bayes estimation based on k-record data under a weighted balanced-type loss function of the form

$$L_{\rho,\omega,\delta_0}(\theta,\delta) = \omega q(\theta)\rho(\delta_0,\delta) + (1-\omega)q(\theta)\rho(\theta,\delta), \qquad (1)$$

where $q(\theta)$ is a suitable positive weighted function, $\rho(\delta_0,\delta)$ is an arbitrary loss function, while δ_0 is a chosen a priori "target" estimator of θ, obtained for instance from the criteria of maximum likelihood, least-squares or unbiasedness among others. The loss function in (1) reflects a desire of closeness of δ to both the target estimator δ_0 and the unknown parameter θ with the relative importance of these criteria governed by the choice of $\omega \in [0,1)$. The aforementioned balanced loss function with $\rho(\delta_0,\delta)$ being a quadratic loss was also used by Jafari Jozani et al. (2006b). Dey et al. (1999) showed that frequentist and Bayesian results for the balanced-type loss function imply related results for quadratic loss functions reflecting only precision of estimation. Here, we shall use $q(\theta) = 1$ and derive the Baysian point estimators for the parameters of interest with some various types of $\rho(\delta_0,\delta)$. In this case, the loss function in (1) may be viewed as a natural extension to Zellner (1994) being specific to a squared error loss (SEL) function $\rho(\delta_0,\delta)$ and a least-squares δ_0.

The rest of the paper is organized as follows: In Section 2, some preliminaries are presented. In Section 3, some main results regarding the prior and posterior distributions of the parameters of interest are presented. The problem of Bayes estimation under three various balanced type loss functions is investigated in Section 4. Furthermore, the Bayesian interval estimation is studied in Section 5. To carry out the performance of the proposed procedure in the paper, a simulation study and sensitivity analysis has been done in Section 6. An illustrative example is presented in Section 7. Eventually, some conclusions are stated.

2 Preliminaries

Let us consider a simple SSALT in which the failure times at level s_i $(i = 1,2)$ are geometrically distributed with successive probability p_i and the corresponding probability mass function (pmf)

$$f_i(x;p_i) = p_i q_i^{x-1}, \qquad x = 1,2,\ldots,$$

where $q_i = 1 - p_i$. Assuming the stress level s_1 is changed to s_2 at the pre-fixed time w_1, it can be shown that the following cumulative exposure distribution (*ced*)

$$G(x) = \begin{cases} G_1(x) = 1 - q_1^x, & x = 1, 2, \ldots, w_1, \\ \\ G_2(x) = 1 - q_1^{w_1} q_2^{x-w_1}, & x = w_1 + 1, w_1 + 2, \ldots, \end{cases} \tag{2}$$

with the corresponding pmf

$$g(x) = \begin{cases} g_1(x) = p_1 q_1^{x-1}, & x = 1, 2, \ldots, w_1, \\ \\ g_2(x) = p_2 q_1^{w_1} q_2^{x-(w_1+1)}, & x = w_1 + 1, w_1 + 2, \ldots. \end{cases}$$

is deduced. Now, suppose that n units are independently placed on a simple SSALT and the experiment is terminated at time w_2 which is corresponding to the Type-I censoring scheme. So, we will observe the data set

$$\mathbf{X}^\star = \{X_{1:n}, \ldots, X_{N_1:n}, X_{N_1+1:n}, \ldots, X_{N_1+N_2:n}, N_1, N_2\},$$

where $X_{i:n}$ stands for the ith smallest order statistic in a sample of size n from the *ced* in (2), N_1 is the number of observations that are less than or equal to w_1 and N_2 denotes the number of data points that are less than or equal to w_2 and are greater than w_1, for which $N_1 + N_2 \leqslant n$. To obtain the joint distribution of the random variables in the data set $\mathbf{X}^\star$, we use the "tie-run" technique in the case of discrete order statistics which is defined by Gan and Bain (1995) regarding the number and lengths of runs of tied observations. Using the concept of tie-run, the joint pmf of the elements of $\mathbf{X}^\star$ is as follows

$$\begin{aligned} f(\mathbf{x}^\star | p_1, p_2) &= c \prod_{i=1}^{n_1} g_1(x_{i:n}) \prod_{i=n_1+1}^{n_1+n_2} g_2(x_{i:n}) \{1 - G_2(w_2)\}^{n-n_1-n_2} \\ &= c\, p_1^{n_1} (1-p_1)^{d_1} p_2^{n_2} (1-p_2)^{d_2}, \end{aligned} \tag{3}$$

where $c = \frac{n!}{(n-n_1-n_2)!} \left(\prod_{j=1}^{r} z_j! \right)^{-1}$ with r being the number of tie-runs with length z_j for the jth one. Moreover, $\mathbf{x}^\star = (x_{1:n}, \ldots, x_{n_1+n_2:n}, n_1, n_2)$ is the observed value of $\mathbf{X}^\star$ and d_1 and d_2 are the observed values of

$$D_1 = \sum_{i=1}^{N_1} X_{i:n} - N_1 + w_1(n - N_1) \tag{4}$$

and

$$D_2 = \sum_{i=N_1+1}^{N_1+N_2} X_{i:n} - (w_1+1)N_2 + (w_2 - w_1)(n - N_1 - N_2), \qquad (5)$$

respectively. Using (3), the maximum likelihood estimator (MLE) of the parameters of interest p_1 and p_2 are obtained as follows

$$\hat{p}_1 = \frac{N_1}{D_1 + N_1} \quad \text{and} \quad \hat{p}_2 = \frac{N_2}{D_2 + N_2}, \qquad (6)$$

which are provided when $N_1 \geqslant 1$ and $N_2 \geqslant 1$, respectively.

3 Main Results

The Bayes approach considers the parameters of interest as random variables with a prior distribution which is constructed by existing information or subjective judgments. As previously mentioned, we assumed that for each level of stress, the life distribution can be described by a geometric distribution where the failure rate is an increasing function of the applied stress levels. Therefore, it is of worthwhile to define a prior distribution over the region specified by $0 < p_1 < p_2 < 1$. In this paper, we consider the Dirichlet distribution as the prior of (p_1, p_2) with the joint probability density function (pdf)

$$\pi(p_1, p_2) = \frac{p_1^{\lambda\alpha_1 - 1}(p_2 - p_1)^{\lambda\alpha_2 - 1}(1 - p_2)^{\lambda\alpha_3 - 1}}{\mathbf{D}(\lambda, \alpha)}, \quad 0 < p_1 < p_2 < 1, \qquad (7)$$

where λ and α_j $(j = 1, 2, 3)$ are known positive constants for which $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and

$$\mathbf{D}(\lambda, \alpha) = \{\Gamma(\lambda)\}^{-1} \prod_{j=1}^{3} \Gamma(\lambda\alpha_j),$$

with $\Gamma(\cdot)$ being the complete gamma function. To obtain the joint posterior of p_1 and p_2, first note that the pmf in (3) can be rewritten via doing some algebraic calculations as

$$f(\mathbf{x}^\star | p_1, p_2) = c \sum_{j=0}^{d_1} \sum_{k=0}^{n_2} \binom{d_1}{j} \binom{n_2}{k} (-1)^{j+k} p_1^{n_1+j} (1 - p_2)^{d_2+k}, \qquad (8)$$

where d_1 and d_2 are the observed values of D_1 and D_2 as defined in (4) and (5), respectively. By using (7) and (8), the joint posterior pdf of p_1 and p_2 is obtained as follows

$$\pi(p_1, p_2|\mathbf{x}^\star) = \frac{1}{I_0}\sum_{j=0}^{d_1}\sum_{k=0}^{n_2}\Bigg\{\binom{d_1}{j}\binom{n_2}{k}(-1)^{j+k} \times p_1^{\lambda\alpha_1+n_1+j-1}(p_2-p_1)^{\lambda\alpha_2-1}(1-p_2)^{\lambda\alpha_3+d_2+k-1}\Bigg\}, \quad (9)$$

where

$$I_0 = \sum_{j=0}^{d_1}\sum_{k=0}^{n_2}\Bigg\{\binom{d_1}{j}\binom{n_2}{k}(-1)^{j+k}B(\lambda\alpha_3+d_2+k,\lambda\alpha_2) \times B(\lambda\alpha_1+n_1+j,\lambda(1-\alpha_1)+d_2+k)\Bigg\}, \quad (10)$$

such that $B(\cdot,\cdot)$ denotes the complete beta function. The marginal posteriors of p_1 and p_2 are achieved by using (9) as

$$\pi_1(p_1|\mathbf{x}^\star) = \frac{1}{I_0}\sum_{j=0}^{d_1}\sum_{k=0}^{n_2}\Bigg\{\binom{d_1}{j}\binom{n_2}{k}(-1)^{j+k}B(\lambda\alpha_3+d_2+k,\lambda\alpha_2) \times p_1^{\lambda\alpha_1+n_1+j-1}(1-p_1)^{\lambda(1-\alpha_1)+d_2+k-1}\Bigg\}, \quad (11)$$

and

$$\pi_2(p_2|\mathbf{x}^\star) = \frac{1}{I_0}\sum_{j=0}^{d_1}\sum_{k=0}^{n_2}\Bigg\{\binom{d_1}{j}\binom{n_2}{k}(-1)^{j+k}B(\lambda\alpha_1+n_1+j,\lambda\alpha_2) \times p_2^{\lambda(1-\alpha_3)+n_1+j-1}(1-p_2)^{\lambda\alpha_3+d_2+k-1}\Bigg\}, \quad (12)$$

respectively, where I_0 is as defined in (10).

Remark 1. When λ tends to 3 and α_i $(i = 1, 2, 3)$ tends to $\frac{1}{3}$, the prior distribution $\pi(p_1, p_2)$ in (7) transforms to the uniform prior as

$$\pi^*(p_1, p_2) = 2, \qquad 0 < p_1 < p_2 < 1, \quad (13)$$

for which the prior in (13) may be considered as a non-informative prior distribution. Therefore, for any given loss function, the Bayes estimators of

p_1 and p_2 in this case can be straightforwardly obtained by performing the aforementioned transformations on those derived for the case of the Dirichlet prior.

4 Bayes Point Estimation

In this section, we study the problem of Bayes point estimation for the parameters p_1 and p_2 under three various balanced type loss functions of the general form (1) with $q(\theta) = 1$. The MLEs of p_1 and p_2 as given in (6) are considered as the associated target estimators of these parameters.

Let us first consider the weighted SEL function $\rho(\theta, \delta) = (\frac{\theta-\delta}{\theta})^2$ in (1), then the balanced loss function

$$L^A_{\omega,\delta_0}(\theta, \delta) = \omega \left(\frac{\delta_0 - \delta}{\delta_0}\right)^2 + (1-\omega)\left(\frac{\theta - \delta}{\theta}\right)^2 \tag{14}$$

can be used to estimate the parameters of interest, where δ_0 is the target estimator of θ. It can be shown that the Bayes estimator of p_i $(i = 1, 2)$ under the balanced weighted SEL in (14) is given by

$$\delta^A_{\omega,p_i}(\mathbf{x}) = \frac{\omega\hat{p}_i^{-1} + (1-\omega)M(p_i; -1)}{\omega\hat{p}_i^{-2} + (1-\omega)M(p_i; -2)}, \tag{15}$$

such that $\hat{p}_i$ is for example the MLE of p_i as given in (6) and $M(\theta; m)$ stands for the posterior expectation of θ^m, i.e.,

$$M(\theta; m) = E(\theta^m|\mathbf{x}^\star)$$

(see, for more details, Jafari Jozani et al., 2006a). Using the marginal posteriors of p_1 and p_2 and some algebraic calculations, we get

$$\begin{aligned} M(p_1; m) = \frac{1}{I_0}\sum_{j=0}^{d_1}\sum_{k=0}^{n_2}\Bigg\{&\binom{d_1}{j}\binom{n_2}{k}(-1)^{j+k}B(\lambda\alpha_3 + d_2 + k, \lambda\alpha_2) \\ &\times B(\lambda\alpha_1 + n_1 + j + m, \lambda(1-\alpha_1) + d_2 + k)\Bigg\}, \end{aligned} \tag{16}$$

where $\lambda\alpha_1 + n_1 > 2$ and

$$\begin{aligned} M(p_2; m) = \frac{1}{I_0}\sum_{j=0}^{d_1}\sum_{k=0}^{n_2}\Bigg[&\binom{d_1}{j}\binom{n_2}{k}(-1)^{j+k}B(\lambda\alpha_1 + n_1 + j, \lambda\alpha_2) \\ &\times B\{(1-\alpha_3)\lambda + n_1 + j + m, \lambda\alpha_3 + d_2 + k\}\Bigg], \end{aligned} \tag{17}$$

provided that $(1-\alpha_3)\lambda + n_1 > 2$.

To study the effect of different loss functions on the estimation of the parameters of interest, we now apply two other loss functions as follows:

- If the entropy loss function $\rho(\theta,\delta) = \frac{\theta}{\delta} - \log(\frac{\theta}{\delta}) - 1$ is considered in (1), the associated balanced loss function is as follows

$$L^B_{\omega,\delta_0}(\theta,\delta) = \omega\left\{\frac{\delta_0}{\delta} - \log\left(\frac{\delta_0}{\delta}\right) - 1\right\} + (1-\omega)\left\{\frac{\theta}{\delta} - \log\left(\frac{\theta}{\delta}\right) - 1\right\}. \tag{18}$$

Using the results derived by Jafari Jozani et al. (2006a), the Bayes estimator of p_i $(i = 1, 2)$ under the balanced entropy loss in (18) is given by

$$\delta^B_{\omega,p_i}(\mathbf{x}) = \omega\hat{p}_i + (1-\omega)M(p_i;1). \tag{19}$$

- The choice $\rho(\theta,\delta) = (\frac{\theta-\delta}{\delta})^2$ in (1), leads to the following balanced loss function

$$L^C_{\omega,\delta_0}(\theta,\delta) = \omega\left(\frac{\delta_0-\delta}{\delta}\right)^2 + (1-\omega)\left(\frac{\theta-\delta}{\delta}\right)^2. \tag{20}$$

The Bayes estimator of p_i $(i = 1, 2)$ under the loss function in (20) is as follows

$$\delta^C_{\omega,p_i}(\mathbf{x}) = \frac{\omega\hat{p}_i^2 + (1-\omega)M(p_i;2)}{\omega\hat{p}_i + (1-\omega)M(p_i;1)}, \tag{21}$$

where $M(p_1;m)$ and $M(p_2;m)$ are as defined in (16) and (17), respectively (see, Jafari Jozani et al., 2006a).

5 Bayesian Interval Estimation

Once the posterior probability density function of the unknown parameter θ is derived, the interval (L,U) is called a $100(1-\alpha)\%$ Bayesian confidence interval for θ, if

$$P(L \leqslant \theta \leqslant U|data) = 1-\alpha. \tag{22}$$

In this section, we obtain the equi-tailed (ET) Bayesian confidence interval and HPD interval for p_1 and p_2.

5.1 Equi-tailed Bayesian Confidence Interval

Using (22), a $100(1-\alpha)\%$ ET Bayesian confidence interval for θ defined on $(0,1)$, can be found by solving the equations

$$\frac{\alpha}{2} = \int_0^L \pi(\theta|data)d\theta \quad \text{and} \quad \frac{\alpha}{2} = \int_U^1 \pi(\theta|data)d\theta,$$

where $\pi(\theta|data)$ stands for the posterior distribution of θ. Therefore, by using the posteriors of p_1 and p_2 given the data set $\mathbf{X}^\star$ presented in (11) and (12), respectively, and performing some algebraic calculations, it can be shown that a $100(1-\alpha)\%$ ET Bayesian confidence interval for p_i $(i=1,2)$ is the solution of the following two equations

$$\frac{\alpha}{2} = \psi_i(L_i) \ \text{ and } \ 1-\frac{\alpha}{2} = \psi_i(U_i), \tag{23}$$

where L_i and U_i are the lower and upper limits of the associated interval, respectively; moreover

$$\begin{aligned}\psi_1(z) = \frac{1}{I_0}\sum_{j=0}^{d_1}\sum_{k=0}^{n_2}\Bigg[&\binom{d_1}{j}\binom{n_2}{k}(-1)^{j+k}B(\lambda\alpha_3+d_2+k,\lambda\alpha_2)\\ &\times B_{ic}\{z;\lambda\alpha_1+n_1+j,\lambda(1-\alpha_1)+d_2+k\}\Bigg],\end{aligned} \tag{24}$$

and

$$\begin{aligned}\psi_2(z) = \frac{1}{I_0}\sum_{j=0}^{d_1}\sum_{k=0}^{n_2}\Bigg[&\binom{d_1}{j}\binom{n_2}{k}(-1)^{j+k}B(\lambda\alpha_1+n_1+j,\lambda\alpha_2)\\ &\times B_{ic}\{z;\lambda(1-\alpha_3)+n_1+j,\lambda\alpha_3+d_2+k\}\Bigg],\end{aligned} \tag{25}$$

where $B_{ic}\{z;a,b\} = \int_0^z t^{a-1}(1-t)^{b-1}dt$ is the incomplete beta function.

5.2 Highest Posterior Density Interval

Here, our objective is to provide the HPD interval for the unknown parameters p_1 and p_2. The HPD interval is one of the most useful tools to measure posterior uncertainty. This interval includes more probable values of the parameter and excludes the less probable ones. If the posterior density of the parameter θ is unimodal, the $100(1-\alpha)\%$ HPD interval (L^*, U^*) for θ must satisfy

$$\int_{L^*}^{U^*} \pi(\theta|data)d\theta = 1-\alpha \quad \text{and} \quad \pi(L^*|data) = \pi(U^*|data),$$

simultaneously. It can be shown that the $100(1-\alpha)\%$ HPD interval (L_i^*, U_i^*) for p_i $(i = 1, 2)$ can be derived by solving the equations

$$\begin{cases} \psi_i(U_i^*) - \psi_i(L_i^*) = 1-\alpha, \\ \\ \pi_i(U_i^*|\mathbf{x}^\star) - \pi_i(L_i^*|\mathbf{x}^\star) = 0, \end{cases} \tag{26}$$

where $\psi_1(z)$ and $\psi_2(z)$ are as defined in (24) and (25), respectively.

6 Simulation Study and Sensitivity Analysis

To carry out the performance of the results obtained in the paper, we present a simulation study and sensitivity analysis in this section. Toward this end, according to the notations of Section 2, we assume that $p_1 = 0.05$, $p_2 = 0.1$, $w_1 = 5$ and $w_2 = 10$. Moreover, we consider three 3-tuples for the prior parameters $\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ as $\tilde{\alpha}_1 = (0.03, 0.02, 0.95)$, $\tilde{\alpha}_2 = (0.05, 0.05, 0.90)$ and $\tilde{\alpha}_3 = (0.10, 0.06, 0.84)$ and three values for the prior parameter λ being 43, 65 and 97. The non-informative prior (the case of $\lambda = 3$ and $\tilde{\alpha}_* = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$) is also considered. Using (15), (19) and (21), the values of the Bayes point estimators δ^A_{ω,p_i}, δ^B_{ω,p_i} and δ^C_{ω,p_i} are obtained, respectively, and the corresponding mean squared errors (MSEs) are computed on the basis of 1000 repetitions of Monte Carlo simulation for $n = 5$ and 10. The results are tabulated in Tables 1 and 2 for the estimation problem of p_1 and p_2, respectively.

Table 1. The values of the Bayes point estimators for p_1 with initial parameters $p_1 = 0.05$ and $p_2 = 0.1$.

			$\omega = 0$			$\omega = 0.25$			$\omega = 0.75$		
n	λ	$\tilde{\alpha}$	δ^A_{ω,p_1}	δ^B_{ω,p_1}	δ^C_{ω,p_1}	δ^A_{ω,p_1}	δ^B_{ω,p_1}	δ^C_{ω,p_1}	δ^A_{ω,p_1}	δ^B_{ω,p_1}	δ^C_{ω,p_1}
5	43	$\tilde{\alpha}_1$	0.01121[1]	0.04254	0.05516	0.01269	0.04499	0.05529	0.02141	0.04988	0.05366
			0.00157[2]	0.00012	0.00010	0.00147	0.00017	0.00015	0.00097	0.00032	0.00038
		$\tilde{\alpha}_2$	0.02270	0.05003	0.06238	0.02539	0.05061	0.06068	0.03632	0.05176	0.05559
			0.00080	0.00006	0.00022	0.00067	0.00012	0.00025	0.00031	0.00033	0.00038
		$\tilde{\alpha}_3$	0.05236	0.07696	0.08851	0.05088	0.07080	0.08275	0.05084	0.05849	0.06575
			0.00007	0.00081	0.00157	0.00011	0.00056	0.00118	0.00028	0.00040	0.00051
	65	$\tilde{\alpha}_1$	0.01734	0.03970	0.04957	0.01936	0.04286	0.05141	0.02938	0.04917	0.05245
			0.00110	0.00014	0.00004	0.00098	0.00015	0.00014	0.00050	0.00031	0.00038
		$\tilde{\alpha}_2$	0.02945	0.04989	0.05949	0.03205	0.05050	0.05862	0.04109	0.05172	0.05494
			0.00045	0.00004	0.00013	0.00036	0.00009	0.00019	0.00019	0.00032	0.00037
		$\tilde{\alpha}_3$	0.06304	0.08175	0.09072	0.05685	0.07439	0.08496	0.05252	0.05969	0.06719
			0.00021	0.00106	0.00171	0.00016	0.00070	0.00130	0.00032	0.00041	0.00054
	97	$\tilde{\alpha}_1$	0.02124	0.03731	0.04479	0.02338	0.04107	0.04826	0.03301	0.04858	0.05152
			0.00084	0.00018	0.00005	0.00073	0.00015	0.00013	0.00033	0.00030	0.00038
		$\tilde{\alpha}_2$	0.03477	0.04985	0.05710	0.03688	0.05047	0.05694	0.04380	0.05171	0.05442
			0.00025	0.00002	0.00007	0.00020	0.00007	0.00015	0.00014	0.00031	0.00036
		$\tilde{\alpha}_3$	0.07203	0.08599	0.09278	0.06086	0.07758	0.08700	0.05347	0.06075	0.06855
			0.00051	0.00133	0.00186	0.00025	0.00084	0.00142	0.00035	0.00042	0.00056
	3	$\tilde{\alpha}_*$	0.03649	0.06699	0.09107	0.04120	0.06332	0.08370	0.06093	0.05600	0.06482
			0.00027	0.00055	0.00200	0.00019	0.00047	0.00144	0.00029	0.00044	0.00059
10	43	$\tilde{\alpha}_1$	0.02057	0.04348	0.05285	0.02291	0.04346	0.05119	0.03200	0.04343	0.04660
			0.00100	0.00013	0.00010	0.00089	0.00018	0.00013	0.00057	0.00033	0.00028
		$\tilde{\alpha}_2$	0.02669	0.04665	0.05568	0.02907	0.04584	0.05346	0.03653	0.04423	0.04753
			0.00064	0.00010	0.00012	0.00055	0.00014	0.00016	0.00039	0.00032	0.00026
		$\tilde{\alpha}_3$	0.04766	0.06545	0.07380	0.04471	0.05994	0.06906	0.04331	0.04893	0.05515
			0.00010	0.00033	0.00066	0.00021	0.00024	0.00047	0.00035	0.00029	0.00022
	65	$\tilde{\alpha}_1$	0.02362	0.04132	0.04903	0.02592	0.04184	0.04837	0.03410	0.04289	0.04564
			0.00077	0.00014	0.00006	0.00067	0.00018	0.00011	0.00042	0.00032	0.00028
		$\tilde{\alpha}_2$	0.03120	0.04705	0.05446	0.03286	0.04614	0.05265	0.03833	0.04433	0.04738
			0.00041	0.00007	0.00008	0.00038	0.00012	0.00014	0.00033	0.00030	0.00025
		$\tilde{\alpha}_3$	0.05645	0.07074	0.07759	0.04885	0.06391	0.07262	0.04435	0.05025	0.05734
			0.00011	0.00050	0.00083	0.00021	0.00031	0.00059	0.00036	0.00027	0.00022
	97	$\tilde{\alpha}_1$	0.02579	0.03921	0.04536	0.02774	0.04026	0.04574	0.03482	0.04237	0.04478
			0.00063	0.00015	0.00006	0.00055	0.00018	0.00011	0.00037	0.00031	0.00029
		$\tilde{\alpha}_2$	0.03522	0.04755	0.05344	0.03581	0.04651	0.05201	0.03952	0.04445	0.04729
			0.00026	0.00004	0.00005	0.00027	0.00010	0.00008	0.00031	0.00029	0.00024
		$\tilde{\alpha}_3$	0.06484	0.07609	0.08154	0.05198	0.06792	0.07635	0.04502	0.05159	0.05969
			0.00026	0.00073	0.00104	0.00026	0.00042	0.00075	0.00037	0.00026	0.00023
	3	$\tilde{\alpha}_*$	0.02791	0.05288	0.06698	0.03104	0.05051	0.06232	0.04168	0.04578	0.05084
			0.00064	0.00023	0.00050	0.00053	0.00025	0.00038	0.00029	0.00035	0.00029

[1] and [2] stand for the Bayes estimator and the corresponding MSE, respectively.

Table 2. The values of the Bayes point estimators for p_2 with initial parameters $p_1 = 0.05$ and $p_2 = 0.1$.

			$\omega = 0$			$\omega = 0.25$			$\omega = 0.75$		
n	λ	$\tilde{\alpha}$	δ^A_{ω,p_2}	δ^B_{ω,p_2}	δ^C_{ω,p_2}	δ^A_{ω,p_2}	δ^B_{ω,p_2}	δ^C_{ω,p_2}	δ^A_{ω,p_2}	δ^B_{ω,p_2}	δ^C_{ω,p_2}
5	43	$\tilde{\alpha}_1$	0.04094[1]	0.06796	0.08109	0.04487	0.07685	0.09018	0.06370	0.09464	0.10047
			0.00360[2]	0.00115	0.00049	0.00317	0.00088	0.00063	0.00161	0.00121	0.00141
		$\tilde{\alpha}_2$	0.07394	0.10102	0.11410	0.07728	0.10164	0.11324	0.08987	0.10290	0.10811
			0.00082	0.00015	0.00035	0.00074	0.00037	0.00056	0.00082	0.00120	0.00124
		$\tilde{\alpha}_3$	0.11020	0.13592	0.14833	0.10390	0.12782	0.14112	0.10193	0.11162	0.12021
			0.00028	0.00148	0.00254	0.00054	0.00119	0.00201	0.00123	0.00135	0.00134
	65	$\tilde{\alpha}_1$	0.04321	0.06378	0.07380	0.04703	0.07371	0.08605	0.06499	0.09359	0.09949
			0.00329	0.00138	0.00076	0.00288	0.00096	0.00071	0.00144	0.00118	0.00142
		$\tilde{\alpha}_2$	0.08044	0.10075	0.11062	0.08222	0.10144	0.11090	0.09201	0.10283	0.10751
			0.00046	0.00008	0.00020	0.00050	0.00029	0.00044	0.00079	0.00116	0.00119
		$\tilde{\alpha}_3$	0.12216	0.14134	0.15065	0.11016	0.13189	0.14346	0.10372	0.11298	0.12178
			0.00060	0.00182	0.00269	0.00069	0.00134	0.00212	0.00131	0.00133	0.00132
	97	$\tilde{\alpha}_1$	0.04503	0.06029	0.06776	0.04869	0.07110	0.08285	0.06577	0.09272	0.09879
			0.00306	0.00162	0.00108	0.00268	0.00106	0.00080	0.00134	0.00115	0.00144
		$\tilde{\alpha}_2$	0.08563	0.10053	0.10781	0.08587	0.10128	0.10905	0.09345	0.10278	0.10278
			0.00025	0.00005	0.00011	0.00037	0.00024	0.00035	0.00077	0.00112	0.00116
		$\tilde{\alpha}_3$	0.13190	0.14592	0.15277	0.11464	0.13532	0.14557	0.10488	0.11413	0.12318
			0.00108	0.00218	0.00285	0.00087	0.00151	0.00224	0.00128	0.00132	0.00131
	3	$\tilde{\alpha}_*$	0.10128	0.17163	0.20516	0.10079	0.15460	0.18905	0.10229	0.12055	0.14183
			0.00115	0.00659	0.01264	0.00127	0.00448	0.00935	0.00161	0.00209	0.00308
10	43	$\tilde{\alpha}_1$	0.05174	0.07039	0.07980	0.05596	0.07668	0.08537	0.07296	0.08927	0.09296
			0.00244	0.00100	0.00053	0.00207	0.00076	0.00049	0.00102	0.00069	0.00068
		$\tilde{\alpha}_2$	0.07657	0.09602	0.10567	0.07925	0.09590	0.10392	0.08819	0.09568	0.09899
			0.00069	0.00016	0.00018	0.00063	0.00026	0.00026	0.00060	0.00059	0.00055
		$\tilde{\alpha}_3$	0.09953	0.11807	0.12725	0.09644	0.11244	0.12136	0.09506	0.10119	0.10611
			0.00016	0.00049	0.00091	0.00031	0.00041	0.00068	0.00063	0.00058	0.00051
	65	$\tilde{\alpha}_1$	0.05168	0.06689	0.07447	0.05575	0.07406	0.08206	0.07239	0.08840	0.09213
			0.00241	0.00118	0.00073	0.00205	0.00085	0.00056	0.00099	0.00068	0.00068
		$\tilde{\alpha}_2$	0.08108	0.09666	0.10435	0.08273	0.09639	0.10307	0.08959	0.09584	0.09882
			0.00045	0.00011	0.00012	0.00046	0.00021	0.00023	0.00056	0.00057	0.00052
		$\tilde{\alpha}_3$	0.11007	0.12483	0.13211	0.10278	0.11751	0.12562	0.09685	0.10288	0.10821
			0.00021	0.00073	0.00115	0.00032	0.00051	0.00082	0.00064	0.00056	0.00049
	97	$\tilde{\alpha}_1$	0.05150	0.06352	0.06948	0.05541	0.07153	0.07911	0.07170	0.08755	0.09142
			0.00240	0.00138	0.00098	0.00205	0.00095	0.00066	0.00099	0.00072	0.00070
		$\tilde{\alpha}_2$	0.08513	0.09725	0.10323	0.08569	0.09683	0.10236	0.09068	0.09599	0.09869
			0.00028	0.00006	0.00007	0.00035	0.00016	0.00016	0.00053	0.00054	0.00050
		$\tilde{\alpha}_3$	0.12004	0.13147	0.13710	0.10807	0.12250	0.13003	0.09816	0.10454	0.11043
			0.00048	0.00107	0.00145	0.00040	0.00067	0.00102	0.00067	0.00055	0.00048
	3	$\tilde{\alpha}_*$	0.09176	0.12958	0.14885	0.09199	0.12108	0.13872	0.09411	0.10407	0.11288
			0.00057	0.00150	0.00307	0.00063	0.00109	0.00215	0.00075	0.00076	0.00083

[1] and [2] stand for the Bayes estimator and the corresponding MSE, respectively.

From Tables 1 and 2, it is observed that:

1. For fixed ω, $\delta^A_{\omega,p_i} < \delta^B_{\omega,p_i} < \delta^C_{\omega,p_i}$ $(i = 1, 2)$.
2. For fixed ω, the choice of the best loss function in view of the MSE criterion depends on the prior parameters α_1, α_2 and α_3, whereas it

seems that λ is not too effective. Moreover, for large values of ω, the sample size is also important. A sensitivity analysis is summarized in Table 3. In this table, the balanced weighted SEL in (14), the balanced entropy loss in (18) and the balanced loss function in (20) are denoted by A, B and C, respectively. Notice that the sensitivity results for estimating p_1 and p_2 are the same.

Table 3. The best loss function for given n, ω and $\tilde{\alpha}$.

	$\omega = 0$		$\omega = 0.25$		$\omega = 0.75$	
$\tilde{\alpha}\backslash n$	5	10	5	10	5	10
$\tilde{\alpha}_1$	C	C	C	C	B	C
$\tilde{\alpha}_2$	B	B	B	B	A	C
$\tilde{\alpha}_3$	A	A	A	A	A	C
$\tilde{\alpha}_*$	A	A	A	A	A	A

From Table 3, it is deduced that for fixed n, ω and given prior parameters, which one of the loss functions imposes minimum MSE. It is also observed that when the effect of the target estimator maximizes, the choice of the best loss function depends on the sample size, whereas for small values of ω, the sample size is not important.

3. For fixed $\tilde{\alpha}$, by choosing a larger value for ω, various loss functions imposes smaller variation of the MSEs of the corresponding Bayes estimators. This issue also holds, when for given loss function, different prior parameters are used. Therefore, by choosing a large value for ω or equivalently maximizing the effect of the target estimator, the sensitivity of the loss function and prior parameters decrease.

For more investigation about the effect of the variation of prior parameters from the case of non-informative prior, a simulation study is also performed for $\lambda = 3$ with initial parameters $p_1 = 0.1$ and $p_2 = 0.15$. We also take $\tilde{\alpha}'_1 = (0.1, 0.05, 0.85)$, $\tilde{\alpha}'_2 = (0.1, 0.1, 0.8)$, $\tilde{\alpha}'_3 = (0.2, 0.1, 0.7)$, $\tilde{\alpha}'_4 = (0.4, 0.2, 0.4)$, $\tilde{\alpha}'_5 = (0.31, 0.32, 0.37)$ and $\tilde{\alpha}_* = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. The results are tabulated in Tables 4 and 5 for the Bayes point estimators of p_1 and p_2, respectively. It is observed that for fixed ω and given loss function, when the prior parameters are close to the case of non-informative prior, the MSEs of the corresponding estimators are also close together.

Table 4. The values of the Bayes point estimators for p_1 with initial parameters $p_1 = 0.1$ and $p_2 = 0.15$.

		$\omega = 0$			$\omega = 0.25$			$\omega = 0.75$		
n	$\tilde{\alpha}$	δ^A_{ω,p_1}	δ^B_{ω,p_1}	δ^C_{ω,p_1}	δ^A_{ω,p_1}	δ^B_{ω,p_1}	δ^C_{ω,p_1}	δ^A_{ω,p_1}	δ^B_{ω,p_1}	δ^C_{ω,p_1}
5	$\tilde{\alpha}'_1$	0.03824[1]	0.09976	0.12691	0.04343	0.09699	0.11921	0.06604	0.09145	0.10108
		0.00446[2]	0.00082	0.00163	0.00394	0.00090	0.00129	0.00215	0.00123	0.00113
	$\tilde{\alpha}'_2$	0.03284	0.09372	0.12203	0.03729	0.09246	0.11488	0.05839	0.08994	0.09896
		0.00513	0.00088	0.00139	0.00466	0.00098	0.00117	0.00282	0.00129	0.00116
	$\tilde{\alpha}'_3$	0.04663	0.10338	0.13062	0.05253	0.09970	0.12222	0.07529	0.09235	0.10230
		0.00342	0.00088	0.00189	0.00290	0.00094	0.00146	0.00145	0.00124	0.00115
	$\tilde{\alpha}'_4$	0.05116	0.11507	0.14150	0.05654	0.10847	0.13159	0.07339	0.09528	0.10665
		0.00336	0.00119	0.00278	0.00298	0.00108	0.00203	0.00206	0.00123	0.00120
	$\tilde{\alpha}'_5$	0.05000	0.10191	0.12909	0.05595	0.09860	0.12076	0.07811	0.09199	0.10144
		0.00304	0.00092	0.00185	0.00255	0.00099	0.00144	0.00127	0.00127	0.00117
	$\tilde{\alpha}_*$	0.05223	0.10363	0.13066	0.05829	0.09989	0.12211	0.08008	0.09242	0.10206
		0.00282	0.00094	0.00195	0.00235	0.00099	0.00151	0.00117	0.00127	0.00118
10	$\tilde{\alpha}'_1$	0.06305	0.09798	0.11225	0.06748	0.09577	0.10718	0.07977	0.09135	0.09571
		0.00188	0.00038	0.00053	0.00160	0.00043	0.00044	0.00100	0.00059	0.00050
	$\tilde{\alpha}'_2$	0.05755	0.09308	0.10802	0.06227	0.09210	0.10367	0.07667	0.09012	0.09426
		0.00230	0.00044	0.00045	0.00196	0.00049	0.00042	0.00116	0.00062	0.00053
	$\tilde{\alpha}'_3$	0.06365	0.09798	0.11264	0.06810	0.09577	0.10742	0.08030	0.09135	0.09574
		0.00179	0.00040	0.00055	0.00152	0.00044	0.00046	0.00094	0.00060	0.00051
	$\tilde{\alpha}'_4$	0.06921	0.10187	0.11646	0.07314	0.09869	0.11051	0.08291	0.09232	0.09697
		0.00138	0.00041	0.00069	0.00118	0.00044	0.00054	0.00082	0.00059	0.00051
	$\tilde{\alpha}'_5$	0.06045	0.09328	0.10811	0.06509	0.09224	0.10369	0.07864	0.09017	0.09423
		0.00198	0.00044	0.00047	0.00166	0.00050	0.00044	0.00099	0.00062	0.00055
	$\tilde{\alpha}_*$	0.06140	0.09403	0.10878	0.06599	0.09281	0.10423	0.07917	0.09036	0.09444
		0.00190	0.00044	0.00048	0.00160	0.00049	0.00045	0.00096	0.00062	0.00054

[1] and [2] stand for the Bayes estimator and the corresponding MSE, respectively.

Table 5. The values of the Bayes point estimators for p_2 with initial parameters $p_1 = 0.1$ and $p_2 = 0.15$.

		$\omega = 0$			$\omega = 0.25$			$\omega = 0.75$		
n	$\tilde{\alpha}$	δ^A_{ω,p_2}	δ^B_{ω,p_2}	δ^C_{ω,p_2}	δ^A_{ω,p_2}	δ^B_{ω,p_2}	δ^C_{ω,p_2}	δ^A_{ω,p_2}	δ^B_{ω,p_2}	δ^C_{ω,p_2}
5	$\tilde{\alpha}'_1$	0.07803[1]	0.13893	0.17489	0.08566	0.14258	0.17151	0.11542	0.14988	0.16289
		0.00582[2]	0.00145	0.00239	0.00488	0.00180	0.00271	0.00267	0.00329	0.00362
	$\tilde{\alpha}'_2$	0.08903	0.15814	0.19779	0.09711	0.15699	0.18931	0.12512	0.15469	0.16999
		0.00452	0.00177	0.00459	0.00374	0.00209	0.00419	0.00247	0.00342	0.00409
	$\tilde{\alpha}'_3$	0.09813	0.16513	0.20390	0.10581	0.16223	0.19445	0.13086	0.15643	0.17242
		0.00353	0.00203	0.00538	0.00295	0.00223	0.00475	0.00237	0.00344	0.00428
	$\tilde{\alpha}'_4$	0.13309	0.21320	0.25339	0.13598	0.19828	0.23675	0.14708	0.16845	0.19310
		0.00183	0.00732	0.01529	0.00202	0.00532	0.01218	0.00300	0.00387	0.00723
	$\tilde{\alpha}'_5$	0.15189	0.23924	0.28018	0.15005	0.21781	0.26049	0.15266	0.17496	0.20551
		0.00203	0.01248	0.02265	0.00238	0.00830	0.01786	0.00343	0.00427	0.00932
	$\tilde{\alpha}_*$	0.15715	0.24496	0.28573	0.15355	0.22211	0.26552	0.15381	0.17639	0.20837
		0.00218	0.01392	0.02452	0.00251	0.00911	0.01938	0.00352	0.00436	0.01000
10	$\tilde{\alpha}'_1$	0.09227	0.12680	0.14624	0.09862	0.13011	0.14509	0.11982	0.13672	0.14201
		0.00372	0.00110	0.00071	0.00309	0.00110	0.00085	0.00172	0.00133	0.00126
	$\tilde{\alpha}'_2$	0.10108	0.13997	0.16152	0.10708	0.13999	0.15659	0.12527	0.14001	0.14594
		0.00287	0.00080	0.00098	0.00240	0.00093	0.00098	0.00155	0.00131	0.00123
	$\tilde{\alpha}'_3$	0.10504	0.14318	0.16433	0.11059	0.14240	0.15883	0.12711	0.14082	0.14681
		0.00250	0.00075	0.00106	0.00212	0.00088	0.00101	0.00148	0.00129	0.00121
	$\tilde{\alpha}'_4$	0.12882	0.17028	0.19244	0.13042	0.16272	0.18169	0.13613	0.14759	0.15597
		0.00110	0.00133	0.00290	0.00116	0.00115	0.00208	0.00136	0.00127	0.00123
	$\tilde{\alpha}'_5$	0.14236	0.18698	0.20990	0.14043	0.17524	0.19647	0.13967	0.15177	0.16249
		0.00087	0.00247	0.00484	0.00104	0.00177	0.00335	0.00138	0.00131	0.00136
	$\tilde{\alpha}_*$	0.14520	0.18990	0.21280	0.14236	0.17743	0.19898	0.14026	0.15250	0.16366
		0.00085	0.00271	0.00522	0.00103	0.00190	0.00360	0.00139	0.00132	0.00139

[1] and [2] stand for the Bayes estimator and the corresponding MSE, respectively.

To compare the ET Bayesian confidence intervals and the HPD intervals, we here present some simulation results for the parameters p_1 and p_2, using the equation systems in (23) and (26), respectively. To this in mind, we take $\lambda = 3$ and use the aforementioned prior parameters $\tilde{\alpha}'_1$, $\tilde{\alpha}'_3$ and $\tilde{\alpha}_*$. The means and variances of the widths of the 95% HPD and ET Bayesian confidence intervals are computed on the basis of 1000 repetitions of Monte Carlo simulation for initial parameters $p_1 = 0.05$ and $p_2 = 0.15$. The results are tabulated in Table 6 for $n = 10$, $w_1 = 5$ and $w_2 = 10$.

Table 6. The values of means and variances of the widths of the 95% HPD and ET Bayesian intervals.

	p_1		p_2	
$\tilde{\alpha}$	ET	HPD	ET	HPD
$\tilde{\alpha}'_1$	0.12910*	0.08041	0.22327	0.10195
	0.00034⋆	0.00277	0.00266	0.01263
$\tilde{\alpha}'_3$	0.12708	0.08253	0.23308	0.14803
	0.00031	0.00267	0.00221	0.01419
$\tilde{\alpha}_*$	0.12746	0.12105	0.25572	0.24990
	0.00028	0.00034	0.00160	0.00168

* and ⋆stand for mean and variance of the Bayesian intervals, respectively.

From Table 6, it is observed that:

1. The expected width of the HPD intervals is less than those of the corresponding ET Bayesian intervals.

2. For the case of non-informative prior ($\tilde{\alpha}_*$), the means and variances of the widths of the HPD and ET Bayesian confidence intervals are closer together than other cases ($\tilde{\alpha}'_1$ and $\tilde{\alpha}'_3$).

7 Illustrative Example

Here, we present a numerical example to illustrate the proposed procedure in the paper. To this in mind, we use the data presented by Xiong (1998) in a simple SSALT, when the failure times in each level of stress have the exponential distribution. It is well-known that if a random variable Y follows an exponential distribution with mean λ^{-1}, then $X = [Y + 1]$ will follow the geometric distribution with parameter $p = 1 - e^{-\lambda}$, where $[z]$ stands for the integer part of z. Therefore, by doing the aforementioned transformation on the data of Xiong (1998), the geometric failure times in each level of stress can be obtained which are summarized in Table 7. We also take $w_1 = 5$.

Table 7. The geometric failure times in a simple SSALT with $w_1 = 5$.

Stress	Failure times										
s_1	3	4	5	5							
s_2	6	6	7	8	8	8	8	9	9	9	9	13

Using the data in Table 7, we would obtain $N_1 = 4$ is the number of units that fail at stress level s_1. Now, we use different choices of w_2 for investigating the variety of inference. For $w_2 = 6, 8$ and 10, the number of units that fail at stress level s_2 are $N_2 = 2, 7$ and 11, respectively. Using (6), the MLE of p_1 is $\hat{p}_1 = 0.041$ and the MLEs of p_2 for $w_2 = 6, 8$ and 10 are $\hat{p}_2 = 0.125, 0.163$ and 0.193, respectively.

To obtain the Bayes estimators of p_1 and p_2, we take $\lambda = 3$ and use the prior parameters $\tilde{\alpha}'_1$, $\tilde{\alpha}'_2$, $\tilde{\alpha}'_3$ and $\tilde{\alpha}_*$ as presented in Section 6. The Bayes point estimators $\delta^A_{\omega,p_i}, \delta^B_{\omega,p_i}$ and δ^C_{ω,p_i} $(i = 1, 2)$ are computed on the basis of the equations in (15), (19) and (21), respectively, for some choices of ω. The results are presented in Table 8 and it is observed that:

Table 8. Values of Bayes point estimators of p_1 and p_2.

			$\omega = 0$			$\omega = 0.25$			$\omega = 0.75$		
$\tilde{\alpha}$	w_2	θ	$\delta^A_{\omega,\theta}$	$\delta^B_{\omega,\theta}$	$\delta^C_{\omega,\theta}$	$\delta^A_{\omega,\theta}$	$\delta^B_{\omega,\theta}$	$\delta^C_{\omega,\theta}$	$\delta^A_{\omega,\theta}$	$\delta^B_{\omega,\theta}$	$\delta^C_{\omega,\theta}$
$\tilde{\alpha}'_1$	6	p_1	0.0258	0.0475	0.0575	0.0313	0.0444	0.0499	0.0355	0.0428	0.0457
		p_2	0.072	0.2706	0.3856	0.0901	0.3853	0.4598	0.1210	0.4426	0.4825
	8	p_1	0.0306	0.0543	0.0646	0.0353	0.0478	0.0545	0.0380	0.0445	0.0484
		p_2	0.0718	0.0958	0.1098	0.0852	0.1053	0.1125	0.0965	0.1100	0.1136
	10	p_1	0.0304	0.0563	0.0670	0.0351	0.0487	0.0561	0.0380	0.0450	0.0493
		p_2	0.0809	0.0983	0.1074	0.0919	0.1047	0.1093	0.1001	0.1079	0.1103
$\tilde{\alpha}'_2$	6	p_1	0.0249	0.0457	0.0557	0.0305	0.0434	0.0488	0.0348	0.0423	0.0451
		p_2	0.1032	0.3186	0.4159	0.1335	0.4093	0.4672	0.1820	0.4546	0.4852
	8	p_1	0.0276	0.0499	0.0596	0.0329	0.0456	0.0513	0.0365	0.0434	0.0465
		p_2	0.0798	0.1055	0.1194	0.0923	0.1101	0.1169	0.1017	0.1124	0.1158
	10	p_1	0.0276	0.0508	0.0609	0.0329	0.0460	0.0521	0.0366	0.0436	0.0469
		p_2	0.0873	0.1045	0.1134	0.0969	0.1078	0.1122	0.1032	0.1094	0.1117
$\tilde{\alpha}'_3$	6	p_1	0.0280	0.0486	0.0585	0.0331	0.0449	0.0506	0.0367	0.0431	0.0461
		p_2	0.1102	0.3336	0.4328	0.1434	0.4168	0.4731	0.1952	0.4584	0.4877
	8	p_1	0.0311	0.0529	0.0624	0.0356	0.0471	0.0532	0.0382	0.0441	0.0475
		p_2	0.0808	0.1058	0.1195	0.0931	0.1103	0.1171	0.1022	0.1125	0.1158
	10	p_1	0.0312	0.0540	0.0637	0.0356	0.0476	0.0539	0.0383	0.0444	0.0480
		p_2	0.0877	0.1046	0.1135	0.0972	0.1078	0.1123	0.1034	0.1095	0.1116
$\tilde{\alpha}_*$	6	p_1	0.0309	0.0504	0.0599	0.0353	0.0458	0.0515	0.0380	0.0435	0.0466
		p_2	0.2867	0.5008	0.5715	0.3545	0.5004	0.5357	0.4110	0.5002	0.5179
	8	p_1	0.0305	0.0489	0.0574	0.0350	0.0450	0.0500	0.0378	0.0431	0.0458
		p_2	0.1043	0.1294	0.1421	0.1093	0.1220	0.1292	0.1120	0.1184	0.1222
	10	p_1	0.0306	0.0489	0.0572	0.0350	0.0450	0.0499	0.0378	0.0431	0.0457
		p_2	0.1040	0.1203	0.1285	0.1074	0.1157	0.1201	0.1092	0.1134	0.1157

1. For fixed ω, $\delta^A_{\omega,p_i} < \delta^B_{\omega,p_i} < \delta^C_{\omega,p_i}$ $(i = 1, 2)$.

2. The value of δ^A_{ω,p_1} increases as ω increases, but δ^B_{ω,p_1} and δ^C_{ω,p_1} are decreasing functions of ω. That is, by maximizing the effect of the MLE of p_1, the corresponding Bayes point estimator under the balanced weighted SEL increases, whereas for two other loss functions, the values of Bayes point estimators decrease.

3. The value of δ^A_{ω,p_2} increases as ω increases. Moreover, δ^B_{ω,p_2} increases [decreases] as ω increases for $\tilde{\alpha}'_i$ $(i = 1, 2, 3)$ $[\tilde{\alpha}_*]$. Furthermore, δ^C_{ω,p_2} is an increasing [a decreasing] function of ω for the prior parameter $\tilde{\alpha}'_1$ $[\tilde{\alpha}_*]$.

The values of 95% ET Bayesian confidence intervals and the HPD intervals are also derived for the parameters p_1 and p_2, using the equation systems in (23) and (26), respectively. The results are presented in Table 9.

Table 9. The values of 95% ET Bayesian and HPD intervals for p_1 and p_2.

		p_1		p_2	
$\tilde{\alpha}$	w_2	*ET*	*HPD*	*ET*	*HPD*
$\tilde{\alpha}'_1$	6	(0.0141, 0.0991)	(0.0101, 0.0909)	(0.0391, 0.6629)	(0.0219, 0.6047)
	8	(0.0164, 0.1051)	(0.0129, 0.0990)	(0.0424, 0.1833)	(0.0356, 0.1734)
	10	(0.0163, 0.1064)	(0.0139, 0.0996)	(0.0506, 0.1661)	(0.0468, 0.1553)
$\tilde{\alpha}'_2$	6	(0.0135, 0.0957)	(0.0096, 0.0880)	(0.0531, 0.6967)	(0.0317, 0.6460)
	8	(0.0148, 0.0989)	(0.0113, 0.0928)	(0.0468, 0.1937)	(0.0403, 0.1823)
	10	(0.0149, 0.1003)	(0.0115, 0.0946)	(0.0546, 0.1725)	(0.0501, 0.1653)
$\tilde{\alpha}'_3$	6	(0.0152, 0.0998)	(0.0112, 0.0921)	(0.0563, 0.7196)	(0.0346, 0.6694)
	8	(0.0168, 0.1022)	(0.0132, 0.0964)	(0.0477, 0.1940)	(0.0412, 0.1825)
	10	(0.0168, 0.1034)	(0.0135, 0.0979)	(0.0552, 0.1724)	(0.0506, 0.1649)
$\tilde{\alpha}^*$	6	(0.0167,0.1011)	(0.0128, 0.0938)	(0.1504, 0.8535)	(0.1485, 0.8515)
	8	(0.0166, 0.0952)	(0.0131, 0.0893)	(0.0630, 0.2199)	(0.0568, 0.2100)
	10	(0.0166, 0.0943)	(0.0133, 0.0888)	(0.0670, 0.1889)	(0.0627, 0.1828)

From Table 9, it is observed that:

(i) The ET Bayesian confidence intervals and the HPD intervals for p_1 and p_2 both contain the corresponding Bayes point estimators with different values of ω and various types of loss functions presented in Section 4.

(*ii*) The widths of the HPD intervals are less than those of the corresponding ET Bayesian confidence intervals.

8 Conclusions

In this paper, a simple step-stress model with two levels of stress was considered in a discrete set-up. It was assumed that the failure times at stress level s_i are geometrically distributed with successive probability p_i $(i = 1, 2)$ and the data are sampled under Type-I censoring scheme. In a Bayesian approach the problems of point and interval estimation of the parameters of interest were investigated. Toward this end, a Dirichlet prior with parameters α_1, α_2, α_3 and λ was considered and by using various balanced loss functions, different point Bayes estimators were derived. Moreover, the HPD and ET Bayesian confidence intervals were obtained. Using a simulation study, it was shown that the choice of the best loss function in view of the MSE criterion depends on the sample size and the prior parameters α_1, α_2 and α_3, whereas it seems that λ is not too effective. Furthermore, it was deduced that the sensitivity of the choice of the loss function and the prior parameters decreases, when the effect of the target estimator increases in the Bayesian estimation problem via choosing large value of ω in a balanced loss function. The simulation results also showed that the expected width of the HPD intervals is less than those of the corresponding ET Bayesian confidence intervals.

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A. Arefi
School of Mathematical Sciences,
Ferdowsi University of Mashhad,
P. O. Box 1159,
Mashhad 91775, Iran.

M. Razmkhah
School of Mathematical Sciences,
Ferdowsi University of Mashhad,
P. O. Box 1159,
Mashhad 91775, Iran.
email: *razmkhah_m@um.ac.ir*

G. R. Mohtashami Borzadaran
School of Mathematical Sciences,
Ferdowsi University of Mashhad,
P. O. Box 1159,
Mashhad 91775, Iran.