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پژوهشکده‌ی آمار

Modified Sampling Strategies Using Correlation Coefficient for Estimating Population Mean

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Abstract. This paper proposes two sampling strategies based on the modified ratio estimator using the population mean of auxiliary variable and population correlation coefficient between the study variable and the auxiliary variable by Singh and Tailor (2003) for estimating the population mean (total) of the study variable in a finite population. A comparative study is made with usual sampling strategies and some concluding remarks are given. Finally, an empirical study is included as an illustration which shows that the proposed sampling strategies are better than Singh and Tailor estimator both in terms of unbiasedness and lesser mean square error.

Keywords. Ratio estimator; unbiasedness; mean square error; range prior information.

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1 Introduction

The use of population correlation coefficient in deriving the efficiency condition of the ratio estimator and product estimator has been discussed by various authors including, Cochran (1977), Sukhatme and Sukhatme (1997) and Singh and Chaudhary (2002). Recently, Singh and Tailor (2003) used the known population correlation coefficient to increase the efficiency of the estimation procedure. Let ρ be the population correlation coefficient between

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the study variable y and the auxiliary variable x. Assuming ρ and population mean $\bar{X}$ of x to be known, Singh and Tailor (2003) proposed modified ratio estimator and modified product estimator for estimating the population mean $\bar{Y}$ of y given by $\bar{y}_{SR} = \bar{y}\big(\bar{X}+\rho\big)/(\bar{x}+\rho)$ and $\bar{y}_{SP} = \bar{y}(\bar{x}+\rho)/\big(\bar{X}+\rho\big)$ respectively; where $\bar{y}$ and $\bar{x}$ be the sample means of the y and x respectively. Singh and Tailor (2003) estimators were found efficient but biased. The expressions of bias and mean square error of $\bar{y}_{SR}$ and $\bar{y}_{SP}$ under Simple Random Sampling without Replacement (SRSWOR) are given by

$$\begin{aligned} \text{Bias}\left(\bar{y}_{SR}\right) &= \gamma_n \bar{Y}\left(\nu^2 C_x^2 - \rho\nu C_x C_y\right) \\ \text{MSE}\left(\bar{y}_{SR}\right) &= \gamma_n \bar{Y}^2\left\{C_y^2 + \nu C_x^2\left(\nu - 2K\right)\right\} \end{aligned} \quad (1)$$

and

$$\begin{aligned} \text{Bias}\left(\bar{y}_{SP}\right) &= \gamma_n \bar{Y}\rho\nu C_x C_y \\ \text{MSE}\left(\bar{y}_{SP}\right) &= \gamma_n \bar{Y}^2\left\{C_y^2 + \nu C_x^2\left(\nu + 2K\right)\right\} \end{aligned} \quad (2)$$

respectively; where $\gamma_n = (1-f_n)/n$, $f_n = n/N$, C_x and C_y are the population coefficient of variation of x and y, respectively, $\nu = \bar{X}/(\bar{X}+\rho)$ and $K = \rho C_y/C_x$.

In this paper, we propose some modified sampling strategies such that the estimator of population mean $\bar{Y}$ is

$$\bar{y}_{SA} = \frac{\bar{y}(\bar{X}+\rho)}{A(\bar{x}+\rho)+(1-A)(\bar{X}+\rho)} \quad (3)$$

where A is a characterizing scalar to be chosen suitably.

This estimator can be seen as the combination of the Walsh (1970) estimator and Singh and Tailor (2003). Note that the proposed estimator reduces to $\bar{y}_{SR}$ if $A = 1$. We now consider this estimator under the following sampling schemes:

1. Simple random sampling without replacement along with the Jack-Knife technique and denote the resulting estimator as $\bar{y}_{SJ}$.

2. Midzuno (1952) – Lahiri (1951) – Sen (1952) type sampling scheme and denote the resulting estimator by $\bar{y}_{SM}$.

Both the sampling strategies aim at getting some classes of better sampling strategies than the existing ones in the sense of unbiasedness and lesser mean square error.

Consider estimator $\bar{y}_{SA}$ under SRSWOR and denote it by $\bar{y}_{SS}$. Let $\bar{y} = \bar{Y} + e_0$ and $\bar{x} = \bar{X} + e_1$ such that

$$E_S(e_0) = E_S(e_1) = 0 \tag{4}$$

where E_S denotes the expectation under SRSWOR.
Putting these values in the estimator and simplifying, we have

$$\bar{y}_{SS} - \bar{Y} = \frac{e_0 - A\bar{Y}e_1}{(\bar{X}+\rho)} + \frac{A^2\bar{Y}^2_{e_1}}{(\bar{X}+\rho)^2} - \frac{Ae_0e_1}{(\bar{X}+\rho)} + \dots \tag{5}$$

Taking expectation on both sides and using (4) we get

$$\text{Bias}\,(\bar{y}_{SS}) = E_s\,(\bar{y}_{SS}) - \bar{Y} = \bar{Y}\left\{\frac{A^2E_s(e_1^2)}{(\bar{X}+\rho)^2} - \frac{AE_s(e_0e_1)}{(\bar{X}+\rho)}\right\}$$

(up to the first order of approximation). Since

$$E_s(e_0^2) = \gamma_n\bar{Y}^2C_y^2, \quad E_s(e_1^2) = \gamma_n\bar{X}^2C_x^2, \quad E_s(e_0e_1) = \gamma_n\bar{X}\bar{Y}\rho C_xC_y \tag{6}$$

Therefore,

$$\text{Bias}\,(\bar{y}_{SS}) = \gamma_n\bar{Y}\left(A^2\nu^2C_x^2 - A\nu\rho C_xC_y\right) = \gamma_n\bar{Y}\left\{A\nu C_x^2\,(A\nu - K)\right\} \tag{7}$$

Now for mean square error consider (7) up to the first order of approximation

$$\begin{aligned}
\text{MSE}(\bar{y}_{SS}) &= E_s(\bar{y}_{SS} - \bar{Y})^2 \\
&= E_s\left\{e_0 - \frac{A\bar{Y}e_1}{(\bar{X}+\rho)}\right\}^2 \\
&= E_s(e_0^2) + \frac{A^2\bar{Y}^2E_s(e_1^2)}{(\bar{X}+\rho)^2} - \frac{2A\bar{Y}E_s(e_0e_1)}{(\bar{X}+\rho)} \\
&= \gamma_n\bar{Y}^2\left(C_y^2 + A^2\nu^2C_x^2 - 2A\nu\rho C_xC_y\right) \\
&= \gamma_n\bar{Y}^2\left\{C_y^2 + \nu C_x^2\left(A^2\nu - 2AK\right)\right\} \qquad (8)
\end{aligned}$$

It is interesting to note that on putting $A = 1$ in (7) and (8) we get expressions of bias and mean square error of $\bar{y}_{SR}$; the optimizing value of the characterizing scalar A is given by $A = K/\nu = A_{opt}$ (say) and the minimum mean square error under optimizing value of $A = A_{opt}$ is

$$\text{MSE}\,(\bar{y}_{SS}) = \gamma_n\bar{Y}^2(1-\rho^2)C_y^2 \tag{9}$$

which is same as the mean square error of the linear regression estimator. Also note that Bias$(\bar{y}_{SS}) = 0$ under the optimizing value of A.

2 The Jackknife Estimator

Let us now apply Quenouille's (1956) method of Jackknife such that the sample of size $n = 2m$ from a population of size N is split up at random into two sub samples of size m each. For further details one may refer to Gray and Schucany (1972). Let us define

$$\bar{y}_{SS}^{(i)} = \frac{\bar{y}_i\left(\bar{X}+\rho\right)}{A\left(\bar{x}_i+\rho\right)+(1-A)\left(\bar{X}+\rho\right)}, \qquad i = 1, 2,$$
$$\bar{y}_{SS}^{(3)} = \frac{\bar{y}\left(\bar{X}+\rho\right)}{A\left(\bar{x}+\rho\right)+(1-A)\left(\bar{X}+\rho\right)} \tag{10}$$

where A is the characterizing scalar to be chosen suitably such that $S = S_1 + S_2$, S_1 and S_2 be the two sub samples of size m each and $+$ denotes the disjoint union. $\bar{y}_1$, $\bar{y}_2$ and $\bar{y}$ denote the sample means based on two sub samples of size m and the entire sample of size $n = 2m$ for characteristic y. $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}$ denote the sample means based on two sub samples of size m and the entire sample of size $n = 2m$ for characteristic x.

It can be easily seen that

$$\text{Bias}\left(\bar{y}_{SS}^{(i)}\right) = \gamma_m\bar{Y}\left\{A^2\nu^2C_x^2 - A\nu\rho C_xC_y\right\}, \qquad i = 1, 2,$$
$$\text{Bias}\left(\bar{y}_{SS}^{(3)}\right) = \gamma_{2m}\bar{Y}\left\{A^2\nu^2C_x^2 - A\nu\rho C_xC_y\right\} = B_1(\text{say}) \tag{11}$$

Let us define $\widehat{\bar{Y}_{SS}}' = (\bar{y}_{SS}^{(1)} + \bar{y}_{SS}^{(2)})/2$ as an alternative estimator of the population mean $\bar{Y}$.

The bias of $\widehat{\bar{Y}_{SS}}'$ is

$$\text{Bias}(\widehat{\bar{Y}_{SS}}') = \gamma_m\bar{Y}\left\{A^2\nu^2C_x^2 - A\nu\rho C_xC_y\right\} = B_2(\text{say}) \tag{12}$$

We propose the Jackknife estimator $\bar{y}_{SJ}$ for estimating population mean $\bar{Y}$ given by

$$\bar{y}_{SJ} = \frac{\bar{y}_{SS}^{(3)} - R\widehat{\bar{Y}_{SS}}'}{1-R} \tag{13}$$

where $R = B_1/B_2$.

Taking expectation of (13) and using (11) and (12) we obtain $E(\bar{y}_{SJ}) = \bar{Y}$ showing that the Jackknife estimator $\bar{y}_{SJ}$ is an unbiased estimator of

population mean $\bar{Y}$ to the first order of approximation.
Consider

$$\begin{aligned}\text{MSE} &= E\left\{\frac{\left(\overset{(3)}{\bar{y}_{SS}} - R\widehat{\bar{Y}_{SS}}'\right)}{(1-R)-\bar{Y}}\right\}^2\\ &= (1-R)^{-2}\Bigg\{E(\overset{(3)}{\bar{y}_{SS}}-\bar{Y})^2 + R^2E(\widehat{\bar{Y}_{SS}}'-\bar{Y})^2\\ &\quad - 2RE(\overset{(3)}{\bar{y}_{ss}}-\bar{Y})(\widehat{\bar{Y}_{SS}}'-\bar{Y})\Bigg\} \qquad (14)\end{aligned}$$

Also,

$$E(\overset{(3)}{\bar{y}_{SS}}-\bar{Y})^2 = \text{MSE}(\overset{(3)}{\bar{y}_{SS}}) = \gamma_{2m}\bar{Y}^2\left\{C_y^2+\nu C_x^2\left(A^2\nu-2AK\right)\right\} \qquad (15)$$

Further,

$$\begin{aligned}E(\widehat{\bar{Y}_{SS}}'-\bar{Y})^2 &= E\left\{\frac{\left(\overset{(1)}{\bar{y}_{SS}}+\overset{(2)}{\bar{y}_{SS}}\right)}{2}-\bar{Y}\right\}^2\\ &= \frac{1}{4}\Bigg\{E(\overset{(1)}{\bar{y}_{SS}}-\bar{Y})^2 + E(\overset{(2)}{\bar{y}_{SS}}-\bar{Y})^2\\ &\quad + 2E(\overset{(1)}{\bar{y}_{SS}}-\bar{Y})(\overset{(2)}{\bar{y}_{SS}}-\bar{Y})\Bigg\} \qquad (16)\end{aligned}$$

Since

$$E(\overset{(i)}{\bar{y}_{SS}}-\bar{Y})^2 = \text{MSE}(\overset{(i)}{\bar{y}_{SS}}) = \gamma_m\bar{Y}^2\left\{C_y^2+\nu C_x^2\left(A^2\nu-2AK\right)\right\}, \quad i=1,2. \qquad (17)$$

Let $\bar{y}_1 = \bar{Y} + e_0^{(i)}$ and $\bar{x}_i = \bar{X} + e_1^{(i)}$ such that $E\left(e_0^{(i)}\right) = E\left(e_1^{(i)}\right) = 0$, $i = 1, 2$. Then, $E(\overset{(1)}{\bar{y}_{SS}}-\bar{Y})(\overset{(2)}{\bar{y}_{SS}}-\bar{Y})$ up to the first order of approximation is

$$E(\overset{(1)}{\bar{y}}_{SS}-\bar{Y})(\overset{(2)}{\bar{y}}_{SS}-\bar{Y}) = E\left\{e_0^{(1)}-\frac{A\bar{Y}e_1^{(1)}}{(\bar{X}+\rho)}\right\}\left\{e_0^{(2)}-\frac{A\bar{Y}e_1^{(2)}}{(\bar{X}+\rho)}\right\}$$
$$= E(e_0^{(1)}e_0^{(2)}) - \frac{A\bar{Y}\left\{E(e_0^{(2)}e_1^{(1)})+E(e_0^{(1)}e_1^{(2)})\right\}}{(\bar{X}+\rho)}$$
$$+\frac{A^2\bar{Y}^2E(e_1^{(1)}e_1^{(2)})}{(\bar{X}+\rho)^2}$$

Substituting the results given in Sukhatme and Sukhatme (1997) $E(e_0^{(1)}e_0^{(2)}) = -\bar{Y}^2C_y^2/N$, $E(e_1^{(1)}e_1^{(2)}) = -\bar{X}^2C_x^2/N$, $E(e_0^{(1)}e_1^{(2)}) = E(e_0^{(2)}e_1^{(1)}) = -\bar{X}\bar{Y}\rho C_xC_y/N$ we have

$$E(\overset{(1)}{\bar{y}}_{SS}-\bar{Y})(\overset{(2)}{\bar{y}}_{SS}-\bar{Y}) = -\frac{\bar{Y}^2\left\{C_y^2+A^2\nu^2C_x^2-2A\nu\rho C_xC_y\right\}}{N}$$
$$= -\frac{\bar{Y}^2\left\{C_y^2+\nu C_x^2\left(A^2\nu-2AK\right)\right\}}{N} \tag{18}$$

Putting the values from (17) and (18) in (16) we have

$$E(\widehat{\bar{Y}_{SS}}'-\bar{Y})^2 = \bar{Y}^2\frac{1}{4}\left[2\left(\frac{1}{m}-\frac{1}{N}\right)-\frac{2}{N}\right]\left\{C_y^2+\nu C_x^2\left(A^2\nu-2AK\right)\right\}$$
$$= \gamma_{2m}\bar{Y}^2\left\{C_y^2+\nu C_x^2\left(A^2\nu-2AK\right)\right\} \tag{19}$$

Now consider

$$E(\overset{(3)}{\bar{y}}_{SS}-\bar{Y})(\widehat{\bar{Y}_{SS}}'-\bar{Y}) = E\left\{\left(\overset{(3)}{\bar{y}}_{SS}-\bar{Y}\right)\left(\frac{\overset{(1)}{\bar{y}}_{SS}+\overset{(2)}{\bar{y}}_{SS}}{2}-\bar{Y}\right)\right\}$$
$$= \frac{1}{2}\left\{E\left(\overset{(3)}{\bar{y}}_{SS}-\bar{Y}\right)\left(\overset{(1)}{\bar{y}}_{SS}-\bar{Y}\right)\right.$$
$$\left.+E\left(\overset{(3)}{\bar{y}}_{SS}-\bar{Y}\right)\left(\overset{(2)}{\bar{y}}_{SS}-\bar{Y}\right)\right\}$$

Since

$$E(\overset{(3)}{\bar{y}_{SS}} - \bar{Y})(\overset{(i)}{\bar{y}_{SS}} - \bar{Y}) = E\left\{e_0 - \frac{A\bar{Y}e_1}{(\bar{X}+\rho)}\right\}\left\{e_0^{(i)} - \frac{A\bar{Y}e_1^{(i)}}{(\bar{X}+\rho)}\right\}$$
$$= E(e_0 e_0^{(i)}) - \frac{A\bar{Y}\left\{E(e_0^{(i)}e_1) + E(e_0 e_1^{(i)})\right\}}{(\bar{X}+\rho)} + \frac{A^2\bar{Y}^2 E(e_1 e_1^{(i)})}{(\bar{X}+\rho)^2}$$

where $i = 1, 2$.
Using the following results given in Sukhatme and Sukhatme (1997) $E(e_0 e_0^{(i)}) = \gamma_{2m}\bar{Y}^2 C_y^2$, $E(e_1 e_1^{(i)}) = \gamma_{2m}\bar{X}^2 C_x^2$, $E(e_0 e_1^{(i)}) = E(e_0^{(i)} e_1) = \gamma_{2m}\bar{X}\bar{Y}\rho C_x C_y$ for $i = 1, 2$, we have

$$E(\overset{(3)}{\bar{y}_{SS}} - \bar{Y})(\overset{(i)}{\bar{y}_{SS}} - \bar{Y}) = \gamma_{2m}\bar{Y}^2\left\{C_y^2 + \nu C_x^2\left(A^2\nu - 2AK\right)\right\}. \tag{20}$$

Putting values from (15), (19), (20) in (14) we have

$$\begin{aligned} \text{MSE}(\bar{y}_{SJ}) &= (1-R)^{-2}\gamma_{2m}\bar{Y}^2(1+R^2-2R)\left\{C_y^2 + \nu C_x^2\left(A^2\nu - 2AK\right)\right\} \\ &= \gamma_n\bar{Y}^2\left\{C_y^2 + \nu C_x^2\left(A^2\nu - 2AK\right)\right\} \end{aligned} \tag{21}$$

which is equal to the mean square error of $\bar{y}_{SS}$. Also, the optimizing value of the characterizing scalar A is given by $A = K/\nu = A_{opt}$ and the minimum mean square error under optimizing value of $A = A_{opt}$ is given by

$$\text{MSE}(\bar{y}_{SJ})_{\min} = \gamma_n\bar{Y}^2(1-\rho^2)C_y^2 \tag{22}$$

which is same as the mean square error of the linear regression estimator. Thus the proposed class of Jackknife sampling strategies is better than the strategy proposed by Singh and Tailor (2003) in the sense that the proposed class of estimators is almost unbiased and possesses an estimator having lesser mean square error.

3 Midzuno – Lahiri – Sen Type Sampling Strategy

Let us consider $\bar{y}_{SA}$ under Midzuno (1952) – Lahiri (1951) – Sen (1952) (MLS) type sampling scheme and denote it by $\bar{y}_{SM}$. The proposed MLS type sampling scheme for selecting a sample S of size n deals with selecting first unit with probability proportional to $\bar{X}+\rho+A(x_i-\bar{X})$ where x_i is the size of the first selected unit such that

$$P(i) = \text{P(selecting first unit } i \text{ with size } x_i = \frac{\left\{\bar{X}+\rho+A(x_i-\bar{X})\right\}}{\left\{N(\bar{X}+\rho)\right\}} \tag{23}$$

and selecting the remaining $n-1$ units in the sample from $N-1$ units in the population by simple random sampling without replacement. Thus the probability of selecting the sample size of n can be expressed as

$$P(s) = \frac{\left\{\bar{X}+\rho+A(\bar{x}-\bar{X})\right\}}{\left\{(\bar{X}+\rho)^N C_n\right\}} \tag{24}$$

Consider

$$\begin{aligned} E(\bar{y}_{SM}) &= E\left[\frac{\bar{y}(\bar{X}+\rho)}{\{\bar{X}+\rho+A(\bar{x}-\bar{X})\}}\right] \\ &= \sum_{s=1}^{{}^N C_n} \frac{\bar{y}(\bar{X}+\rho)P(s)}{\left\{\bar{X}+\rho+A(\bar{x}-\bar{X})\right\}} \\ &= \sum_{s=1}^{{}^N C_n} \frac{\bar{y}}{{}^N C_n} = E_s(\bar{y}) = \bar{Y} \end{aligned} \tag{25}$$

showing that $\bar{y}_{SM}$ is an unbiased estimator of population mean $\bar{Y}$ for all values of A under the proposed MLS type sampling scheme.
Now, in order to obtain MSE of the proposed class of estimators, let us consider

$$\begin{aligned} E(\bar{y}^2_{SM}) &= \sum_{s=1}^{{}^N C_n} \bar{y}^2_{SM}.P(s) \\ &= \sum_{s=1}^{{}^N C_n} \frac{\bar{y}^2(\bar{X}+\rho)}{\left\{\bar{X}+\rho+A(\bar{x}-\bar{X})\right\}{}^N C_n} \\ &= E_s\left[\left(\bar{Y}+e_0\right)^2\left\{1+\frac{Ae_1}{(\bar{X}+\rho)}\right\}^{-1}\right] \\ &= \bar{Y}^2+E_s(e_0^2)+\frac{A^2\bar{Y}^2E_s(e_1^2)}{(\bar{X}+\rho)^2}-\frac{2A\bar{Y}E_s(e_0e_1)}{\left(\bar{X}+\rho\right)} \end{aligned}$$

(upto 1st order of approx). Therefore by using (6) we have

$$\begin{aligned}\text{MSE}(\bar{y}_{SM}) &= \gamma_n \bar{Y}^2 \left\{C_y^2 + \nu C_x^2 \left(A^2\nu - 2AK\right)\right\}\\ &= \text{MSE}(\bar{y}_{SS}) = \text{MSE}(\bar{y}_{SJ}) = \text{MSE}(\bar{y}_{SA}) \quad \text{(say)} \qquad (26)\end{aligned}$$

The optimizing value of the characterizing scalar A is given by $A = K/\nu = A_{opt}$ and the minimum mean square error under optimizing value of $A = A_{opt}$ is

$$\begin{aligned}\text{MSE}(\bar{y}_{SM})_{\min} &= \gamma_n \bar{Y}^2 (1-\rho^2) C_y^2\\ &= \text{MSE}(\bar{y}_{SS})_{\min} = \text{MSE}(\bar{y}_{SJ})_{\min} = \text{MSE}(\bar{y}_{SA})_{\min} \quad \text{(say)} \qquad (27)\end{aligned}$$

which is same as the mean square error of the linear regression estimator. Thus the proposed class of MLS type sampling strategies is better than the strategy proposed by Singh and Tailor (2003) in the sense that the proposed class of estimators is unbiased and possesses an estimator having lesser mean square error.

4 Concluding Remarks

If the minimizing value $A = K/\nu = A_{opt}$ is known then, we have

$$\text{MSE}(\bar{y}) - MSE(\bar{y}_{SA})_{\min} = \gamma_n \bar{Y}^2 \rho^2 C_y^2 \geqslant 0$$

$$\text{MSE}(\bar{y}_R) - \text{MSE}(\bar{y}_{SA})_{\min} = \gamma_n \bar{Y}^2 \left(C_x - \rho C_y\right)^2 \geqslant 0$$

$$\text{MSE}(\bar{y}_P) - \text{MSE}(\bar{y}_{SA})_{\min} = \gamma_n \bar{Y}^2 \left(C_x + \rho C_y\right)^2 \geqslant 0$$

$$\text{MSE}(\bar{y}_{SR}) - \text{MSE}(\bar{y}_{SA})_{\min} = \gamma_n \bar{Y}^2 \left(\nu C_x - \rho C_y\right)^2 \geqslant 0$$

$$\text{MSE}(\bar{y}_{SP}) - \text{MSE}(\bar{y}_{S})_{\min} = \gamma_n \bar{Y}^2 \left(\nu C_x + \rho C_y\right)^2 \geqslant 0$$

where $\bar{y}_R = \bar{y}\bar{X}/\bar{x}$, $\bar{y}_P = \bar{y}\bar{x}/\bar{X}$ and $\bar{y}_{lr} = \bar{y} + b(\bar{X} - \bar{x})$ are classical ratio, product and linear regression estimators respectively.

Hence, under the optimizing value of the characterizing scalar $A = K/\nu = A_{opt}$, the mean square error of the proposed sampling strategies is always lesser than that of $\bar{y}$, $\bar{y}_{SR}$, $\bar{y}_{SP}$, $\bar{y}_P$ and $\bar{y}_R$. Therefore, the proposed sampling strategies are always better than $\bar{y}$, $\bar{y}_{SR}$, $\bar{y}_{SP}$, $\bar{y}_P$, $\bar{y}_R$ and $\bar{y}_{lr}$ both in sense of unbiasedness and gain in efficiency.

Table 1. Bias, Mean Square Error and Percent Relative Efficiency

Estimator(t)	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}_{lr}$	$\bar{y}_{SP}$	$\bar{y}_{SR}$	$\frac{\bar{y}_{SM}}{\bar{y}_{SJ}}$
Bias (t)	0	-244.68	3376.77	3867.58	3247.27	-350.81	0
MSE (t)	6564590	1249925	21072230	1221872	20346316	1284228	1221872
PRE(t:$\bar{y}$)	100	525.20	31.15	537.26	32.26	511.1697	537.26

5 Empirical Study

Here an empirical study has been carried out on an example considered by Singh and Chaudhary (1986) is given, wherein the following values are obtained $\bar{Y} = 1467.545$, $\bar{X} = 22.62$, $C_x = 1.460904$, $C_y = 1.7459$, $\rho = 0.9022$. The bias, mean square error and percent relative efficiency (PRE) of the sample mean $\bar{y}$, ratio estimator $\bar{y}_R$, product estimator $\bar{y}_P$, $\bar{y}_{SP}$, $\bar{y}_{SR}$ and $\bar{y}_{lr}$ are given in Table 1.

It should be noted that the above results are scaled by the factor γ_n. Further, it can be easily observed from the table that only sample mean $\bar{y}$ and the proposed sampling strategies are unbiased. Further, it can be seen that $\bar{y}_{lr}$ and the proposed sampling strategies attains the minimum mean square error but $\bar{y}_{lr}$ is biased. It is evident from the above empirical study that under the optimizing value of the proposed sampling strategies are better than the remaining sampling strategies both in terms of unbiasedness and mean square error.

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