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Dispersive Ordering and k-out-of-n Systems

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Extended Abstract. The simplest and the most common way of comparing two random variables is through their means and variances. It may happen that in some cases the median of X is larger than that of Y, while the mean of X is smaller than the mean of Y. However, this confusion will not arise if the random variables are stochastically ordered. Similarly, the same may happen if one would like to compare the variability of X with that of Y based only on numerical measures like standard deviation etc. Besides, these characteristics of distributions might not exist in some cases. In most cases one can express various forms of knowledge about the underlying distributions in terms of their survival functions, hazard rate functions, mean residual functions, quantile functions and other suitable functions of probability distributions. These methods are much more informative than those based only on few numerical characteristics of distributions. Comparisons of random variables based on such functions usually establish partial orders among them. We call them as stochastic orders.

Stochastic models are usually sufficiently complex in various fields of statistics, particularly in reliability theory. Obtaining bounds and approximations for their characteristics is of practical importance. That is, the approximation of a stochastic model either by a simpler model or by a model with simple constituent components might lead to convenient bounds and approximations for some particular and desired characteristics of the model. The study of changes in the properties of a model, as the constituent components vary, is also of great interest. Accordingly, since the stochastic components of models involve random variables, the topic of stochastic orders among random variables plays an important role in these areas.

In this paper, the properties of dispersive ordering are discussed and some of the results obtained recently in the area of stochastic comparisons of order statistics and sample spacings are reviewed. We consider the cases when the parent observations are identically as well as non-identically distributed. But most of the time we shall be assuming that the observations are independent.

Let $X_{i:n}$ denote the ith order statistic of a random sample of size n from a continuous distiburion function F. Let $Y_{j:m}$ denote the jth order statistic of a random sample of size m from a continuous distribution function G. It is shown that if X is less dispersed than Y and F or G is DFR, then for $i \leqslant j$ and $n-i \geqslant m-j$, $X_{i:n}$ is less than $Y_{j:m}$ in the sense of dispersive ordeing as well as hazard rate ordering. Also, if X_i's are independent random variables, but not identically distributed, then $X_{i:n}$ is less dispersed than $X_{j:n}$, for $i \leqslant j$, provided $X_{i:n}$ is DFR. In particular, if X_i's are also DFR, then it is proved that $X_{1:n}$ is less dispersed than $X_{j:n}$, for $j > 1$.

Now, let $W_1, \ldots, W_n$ be independent random variables with W_i having hazard rate $\lambda_i h(t)$, $i = 1, \ldots, n$. Let $V_1, \ldots, V_n$ be a random sample of size n from a distribution with common hazard rate $\tilde{\lambda} h(t)$, where $\tilde{\lambda} = (\prod_{i=1}^{n} \lambda_i)^{\frac{1}{n}}$, the geometric mean of the λ_i's. Let $W_{n:n} = \max\{X_1, \ldots, X_n\}$. It is shown that $W_{n:n}$ is greater than $V_{n:n}$ according to hazard rate ordering and if the baseline distribution is DFR, then $W_{n:n}$ is greater than $V_{n:n}$ according to dispersive ordering. These results are extensions of those proved in Khaledi and Kochar (J. Appl. Prob. (2000) and lead to a lower bound for the variance of $W_{n:n}$ and an upper bound on the hazard rate function of $V_{n:n}$ in terms of $\tilde{\lambda}$.

Let $Z_1, \ldots, Z_n$ be independent exponential random variables with possibly different scale parameters and $D_{i:n} = (n-i+1)(Z_{i:n} - Z_{i-1:n})$ denote the corresponding ith normalized spacing. Here $Z_{0:n} \equiv 0$. Kochar and Korwar (1996) conjectured that in this case the successive normalized spacings are increasing according to hazard rate ordering. This conjecture is proved this conjecture in the case of a single-outlier exponential model when all parameters, except one, are identical. We also prove that the spacings are more dispersed and larger in the sense of hazard rate ordering when the vector of scale parameters is more dispersed in the sense of majorization.

Keywords. usual stochastic order; hazard rate order; likelihood ratio order; majorization; p-larger; schur functions; proportional hazard models; k-out-of-n systems; spacings.

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