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پژوهشکده‌ی آمار

# Inference for a Skew Normal Distribution Based on Progressively Type-II Censored Samples

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**Extended Abstract.** In many industrial experiments involving lifetimes of machines or units, experiments have to be terminated early or the number of experiments must be limited due to a variety of circumstances (e.g. when expensive items must be destroyed, when experiments are time- consuming and expensive, etc.). The samples that arise from such experiments are called censored data.

Cohen (1991) was one of the earliest to study a more general censoring scheme called progressive censoring scheme. The progressive Type-II censoring scheme, after starting the life-testing experiment with $n$ units, can be described as follows. Immediately following the first failure, $R_1$ surviving units are removed from the test at random. Then, immediately following the second observe failure, $R_2$ surviving units are removed from the test at random. This process continues until, at the time of the $m$-th observed failure, all the remaining $R_m = n - R_1 - R_2 - \cdots - R_{m-1} - m$ units are removed from the experiment. Here the $R_i$'s are fixed prior to study. If $R_1 = R_2 = \cdots = R_m = 0$, then $n = m$ which corresponds to the complete sample situation. If $R_1 = R_2 = \cdots = R_{m-1} = 0$, we have $R_m = n - m$ which corresponds to the conventional Type-II right censoring scheme.

Let the failure time distribution be a skew normal distribution with probability density function (pdf)

$$f(y, \mu, \sigma, \lambda) = 2\phi\left(\frac{y-\mu}{\sigma}\right)\Phi\left\{\lambda\left(\frac{y-\mu}{\sigma}\right)\right\}, \qquad -\infty < y < +\infty$$

where $-\infty < \mu < +\infty$ and $\sigma > 0$ are the location and scale parameters, respectively, and $-\infty < \lambda < +\infty$ is the shape parameter. Here $\phi(\cdot)$ and

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$\Phi(\cdot)$ are the $N(\mu, \sigma^2)$ density and cumulative distribution function, respectively. When $\lambda = 0$, the above distribution becomes the $N(\mu, \sigma^2)$ distribution. This distribution was introduced by Azzalini (1985) and was extensively discussed by Azzalini (1986), Henze (1986), Azzalini and Dalla-Valle (1996), and Arnold and Beaver (2002).

For the above skew normal distribution, the maximum likelihood method do not provide explicit estimators for the location and scale parameters based on a progressively Type-II censored sample. This paper provides a simple method of deriving explicit estimators by approximating the likelihood function. We study the bias and variance of the maximum likelihood estimators (MLEs) and the approximate estimators and show that the approximation provides estimators that are almost as efficient as the MLEs. Next, we show that the probability coverages of the pivotal quantities (for the location and scale parameters) based on asymptotic normality are unsatisfactory, especially when the effective sample size is small. Therefore, we suggest using unconditional simulated percentage points of these pivotal quantities for the construction of confidence intervals. Finally, we present two numerical examples to illustrate all the methods of inference discussed here.

**Keywords.** confidence interval; Fisher information; maximum likelihood estimator; Monte Carlo simulation; pivotal quantity; progressive Type II censoring; pivotal quantity; skew normal distribution.

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