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پژوهشکده‌ی آمار

Skew Normal State Space Modeling of RC Electrical Circuit and Parameters Estimation based on Particle Markov Chain Monte Carlo

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Abstract. In this paper, a skew normal state space model of RC electrical circuit is presented by considering the stochastic differential equation of the this circuit as the dynamic model with colored and white noise and considering a skew normal distribution instead of normal as the measurement noise distribution. Optimal filtering technique via sequential Monte Carlo perspective is developed for tracking the charge as the hidden state of this model. Furthermore, it is assumed that this model contains unknown parameters (resistance, capacitor, mean, variance and shape parameter of the skew normal as the measurement noise distribution). Bayesian framework is applied for estimation of both the hidden charge and the unknown parameters using particle marginal Metropolis-Hastings scheme. It is shown that the coverage percentage of skew normal is more than the one of normal as the measurement noise. Some simulation studies are carried out to demonstrate the efficiency of the proposed approaches.

Keywords. RC electrical circuit; state space model; sequential Monte Carlo filtering; parameter estimation.

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1 Introduction

Time varying systems such as electrical circuits can be described in different ways using state space models (Yarlagadda, 1972; Vongpanitlerd, 1970). A state space realisation for a circuit provides more insight into the topology of the circuit (Anderson and Vongpanitlerd, 1973). In these models that are commonly used in control, signal processing, Econometrics, robotics and telecommunications, the state of the system is indirectly observed through noisy observation. The state of the system contains information required to describe the system under investigation. The key purpose in these systems is optimal filtering, which refers to the method that can be used for estimating the state of a time varying system from noisy observation. The Kalman filter method in linear Gaussian state space models and its modified methods like the Extended Kalman filter (EKF), the Unscented Kalman filter (UKF) and Gaussian filter have been applied in non-liner Gaussian state space models (Sarkka, 2007; Julier and Uhlmann, 2004; Wu et al., 2006). Sequential Monte Carlo (SMC) methods, which provide very good approximations to the optimal filter under weak assumptions for non-linear non-Gaussian state space models, have been studied in many recent researches (Kantas et al., 2009; Andrieu et al., 2010; Doucet et al., 2000). The main idea of SMC methods is to represent the approximation of filtering distribution through the techniques, the importance sampling (IS), the sequential importance sampling (SIS) and the sequential importance resampling (SIR) (Doucet et al., 2000; Liu, 2001; Kitagawa, 1998). In many applicable situations, the state space models contain the unknown parameters that need to be estimated from the data. To date, several works have been presented on the static parameter estimation problem from Bayesian and Maximum likelihood (ML) methods using off-line and on-line framework in the state space models. The ML method by using two mentioned frameworks with a gradient approach has been studied in Coquelin et al. (2008), Poyadjis et al. (2009). Parameter estimation in ML approach via Expectation-Maximization optimization in the general state space models in the off-line and on-line frameworks has been proposed in Andrieu et al. (2004), Wills et al. (2008). Bayesian paradigms using informative priors regarding to the unknown parameters are flexible approaches to assess parameters in a large class of these models. The particle Markov chain Monte Carlo (PMCMC) based on SMC as a standard approach for joint estimation of the static parameter and hidden state has been studied in Andrieu et al. (2010). In on-line framework of Bayesian method, the artificial dynamics approach and resample move have been presented in works of (Kitagawa, 1998; Fearnhead, 2002). Particle approximations of the score and observed information matrix and parameter estimation in state space models have been studied in Poyiadjis et al. (2011). Campillo and Rossi (2009) proposed a variant of the particle filter based on convolution kernel approximation techniques for parameter estimation in general state space models. Parameter estimation for a time varying non-linear circuit from state analysis and simulation has been investigated in Vats and Parthasarathy (2008).

Recently, parameter estimation of RL electrical circuit based on least Square and Bayesian approach has been presented by Farnoosh et al. (2012). Farnoosh and Hajrajabi, (2013) considered the state space modeling of stochastic RL electrical circuit and parameter estimation via ML and Bayesian approach. In this paper, the mathematical model of the RC electrical circuit as a series resistor-capacitor circuit is considered as stochastic differential equation (SDE) by adding some randomness to the ordinary differential equation. The SDE of the RC electrical circuit with colored and white noise is assumed as the dynamic model of a state space system. If the charge in the circuit is corrupted by the measurement noise, on the other hand is hidden, it will be considered as the state of the system. In empirical situation, the measurement noise tends to be much more skewed and has a much higher kurtosis than is allowed by a normal distribution. It is assumed that the measurement noise has a skew normal distribution. This allows a flexible treatment of asymmetry and heavy tails in the distribution of measurement noise. We are interested in tracking the hidden state from noisy observation using the SMC method in skew normal state space model of the RC electrical circuit (SNSS-RC). Furthermore, it is assumed that this model contains the unknown parameters (resistance, capacitor, mean, variance and shape parameter of the skew normal distribution) and these parameters are estimated from the Bayesian approach based on PMCMC scheme. To the best of the author's knowledge, the problem of considering the skew normal state space model of this circuit, tracking the hidden charge as the hidden state via optimal filtering based on SMC scheme and estimation of the unknown parameters based on PMCMC have not been studied before.

The structure for the remainder of the article is as follows. In Section 2, the skew normal state space model of the RC electrical circuit is presented by considering the SDE of this circuit as the dynamic model. Section 3, covers optimal filtering for estimation of the hidden charge in the model via SMC method. In Section 4, based on the model, PMCMC procedure for Bayesian estimation is derived. In Section 5, the numerical experiments are conducted to verify the accuracy of proposed methods. Some conclusions are given in Section 6.

2 Skew Normal State Space Modeling of the RC Electrical Circuit

When a circuit contains only a charged capacitor and a resistor, a series resistor-capacitor (RC) circuit is obtained. The mathematical model of this circuit is written by Kirchhoff's voltage law as below,

$$\frac{dQ(t)}{dt} = -\frac{1}{RC}Q(t) + \frac{1}{R}V(t), \qquad Q(0) = Q_0,$$

where Q(t), V(t) denote the charge on the capacitor at time t and the potential source applied to the input of a RC circuit. Q_0 is the arbitrary and known initial value of Q(t) at time 0. By adding some randomness in the potential source, the SDE of this model is derived as follows:

$$\frac{dQ(t)}{dt} = -\frac{1}{RC}Q(t) + \frac{1}{R}\{V(t) + \alpha\epsilon(t)\}, \qquad Q(0) = Q_0, \tag{1}$$

α is intensity of noise and $\epsilon(t)$ can be considered as the white noise if it is a derivation of winer process $W(t)$ such that $W(0) = 0\ a.s$ and $W(t) - W(s) \sim N(0, t-s)$, for all $t \geqslant s \geqslant 0$ and as the colored noise if it is an Orstein-Uhlenbeck process. By replacing the corresponding linear additive SDE of mentioned noise, $dX(t) = dW(t)$ and $dX(t) = \mu X(t)dt + \sigma dW(t)$ with μ and σ as drift and diffusion coefficient, the equation (1) can be written as,

$$dQ(t) = -\frac{1}{RC}\{Q(t) - CV(t)\}dt + \frac{\alpha}{R}dX(t), \qquad Q(0) = Q_0. \tag{2}$$

For a given positive integer n that is assumed the number of observations, let time step $\Delta t = \frac{T}{n}$ and consider the partitions $\{0, \Delta t, 2\Delta t, \ldots, (n-1)\Delta t, T\}$ of the interval $[0, T]$ that T is chosen arbitrary and for simplicity consider that $V(t)$ has a constant value in this interval. The following discrete SDE can be derived with a simple forward Euler discretization of the equation (2), (Oksendal, 2000)

$$\begin{aligned} Q_{t+\Delta t} &= Q_t - \frac{1}{RC}(Q_t - CV)\Delta t + \frac{\alpha}{R}(X_{t+\Delta t} - X_t), \\ & \quad t = (k-1)\Delta t, \qquad k = 1, 2, \ldots, n, \end{aligned} \tag{3}$$

where $X_{t+\Delta t}$ denote the white and colored noise and also the following discrete SDEs can be derived for them respectively as,

$$\begin{aligned} &X_{t+\Delta t} = W_{t+\Delta t}, \\ &X_{t+\Delta t} = X_t + \mu X_t \Delta t + \sigma\sqrt{\Delta t} W_{t+\Delta t}, \quad t = (k-1)\Delta t, \quad k = 1, 2, \ldots, n. \end{aligned}$$

Another form of the equation (4) is obtained by denoting $Q_{k\Delta t}$ and $X_{k\Delta t}$ by Q_k and X_k respectively

$$Q_k = Q_{k-1} - \frac{1}{RC}(Q_{k-1} - CV)\Delta t + \frac{\alpha}{R}(X_k - X_{k-1}), \quad k = 1, 2, \ldots, n.$$

When the charge at time k is not directly visible and is only observed through noisy measurement, the discrete state space model of RC electrical circuit can be obtained as,

$$Q_k = Q_{k-1} - \frac{1}{RC}(Q_{k-1} - CV)\Delta t + \frac{\alpha}{R}(X_k - X_{k-1}), \tag{4}$$

$$y_k = Q_k + r_k, \quad k = 0, \ldots, n. \tag{5}$$

The models (4) and (5) denote the dynamic and the measurement model. Where y_k, Q_k and r_k are the observation, the hidden state of the model and the measurement noise at discrete time k, respectively. When the measurement noise has skewness and heavy tails, there is a general tendency to find more flexible models to present the features of this process. By considering skew normal distribution (SN) denoted $r_k \sim SN(\mu_r, \sigma_r^2, \lambda_r)$ and introduced by Azzalini (2005) , the density of measurement noise is written as below,

$$f(r_k) = \frac{2}{\sigma_r}\phi\left(\frac{r_k - \mu_r}{\sigma_r}\right)\Phi\left\{\frac{\lambda_r(r_k - \mu_r)}{\sigma_r}\right\},$$

where ϕ, Φ, μ_r, σ_r^2, λ_r denote the density of normal, the cumulative distribution of normal, the mean, the variance and the shape parameter of SN distribution, respectively.

The conditional density of Q_k provided $Q_{k-1}, R, C, V, \alpha, \Delta t, X_k, X_{k-1}$ given in equation (4) is normally distributed,

$$p(Q_k|Q_{k-1}, R, C, V, \epsilon, \Delta t, X_k, X_{k-1}) \sim N(Q_{k-1} - \frac{1}{RC}(Q_{k-1} - CV)\Delta t,$$
$$+ \frac{\alpha}{R}\mu_{X_k}(\frac{\alpha}{R})^2\sigma^2_{X_k}),$$

where μ_{X_k} and $\sigma^2_{X_k}$ are the mean and the variance of mentioned measurement noise processes. The conditional density of y_k can be written as

$$p(y_k|Q_k, \mu_r, \sigma_r^2, \lambda_r) \sim SN(\mu_r + Q_k, \sigma_r^2, \lambda_r).$$

SNSS-RC model can be derived as follow,

$$\begin{cases} p(Q_k|Q_{k-1}, R, C, \alpha, \Delta t, X_k, X_{k-1}) \sim N(Q_{k-1} - \frac{1}{RC}(Q_{k-1} - CV)\Delta t \\ +\frac{\alpha}{R}\mu_{X_k}, (\frac{\alpha}{R})^2\sigma^2_{X_k}), \\ p(y_k|Q_k, \mu_r, \sigma_r^2, \lambda_r) \sim SN(\mu_r + Q_k, \sigma_r^2, \lambda_r) \quad k = 1, \dots, n. \end{cases} \tag{6}$$

3 Tracking the Hidden Charge Via SMC

From a Bayesian perspective, the first area of interest is to characterize the distribution of the hidden state Q_k of the SNSS-RC model at the present time k, given the information provided by all of the observations received up to the present time, $y_{1:k} = (y_1, \dots, y_k)$. Hence, it is required to construct the density $p(Q_k|y_{1:k})$ denoted the filtering distribution that can be used to estimate any property of $p(Q_k|y_{1:k})$ in an ordinary Monte Carlo estimation framework. It is assumed that the initial density $p(Q_0|y_0) = p(Q_0)$ of the state vector, which is also known as the prior, is available. The main purpose is to express the required density function by a set of random samples with associated weights and to compute estimates based on these samples and weights (Andrieu et al., 2010; Doucet, 2000). The SMC scheme uses a weighted set of particles $\{w^i_{0:k}, Q^i_{0:k}\}_{i=1}^N$ for representing an approximation of the $p(Q_{0:k}|y_{1:k})$ as,

$$p(Q_{0:k}|y_{1:k}) \approx \sum_{i=1}^N w_k^i \delta(Q_{0:k} - Q^i_{0:k}), \qquad \sum_{i=1}^N w_k^i = 1, \tag{7}$$

where $\delta(\cdot)$ is the Dirac delta function and for each particle, w_k^i is proportional to

$$w_k^i \propto \frac{p(Q_{0:k}^i|y_{1:k})}{q(Q_{0:k}^i|y_{1:k})}, \tag{8}$$

where $q(Q_{0:k}|y_{1:k})$ is an importance density that the samples $Q_{0:k}^i$ are drawn from it. The key idea is to factorize distributions p and q for obtaining the weight w_k^i. If the importance density is chosen to factorize such that

$$q(Q_{0:k}|y_{1:k}) = q(Q_k|Q_{0:k-1}, y_{1:k})q(Q_{0:k-1}|y_{1:k-1}), \tag{9}$$

then the samples $Q_{0:k}^i$ can be obtained from augmenting each of the existing samples $Q_{0:k-1}^i \sim q(Q_{0:k-1}|y_{1:k-1})$ with the new state $Q_k^i \sim q(Q_k|Q_{0:k-1}, y_{1:k})$. To derive the weight equation, $p(Q_{0:k}|y_{1:k})$ is written in terms of the following densities as below,

$$\begin{aligned} p(Q_{0:k}|y_{1:k}) &= \frac{p(y_k|y_{1:k-1}, Q_{0:k})p(y_{1:k-1}|Q_{0:k})p(Q_{0:k})}{p(y_k|y_{1:k-1})p(y_{1:k-1})} \\ &= \frac{p(y_k|y_{1:k-1}, Q_{0:k})p(Q_k|Q_{0:k-1})p(Q_{0:k-1}|y_{1:k-1})}{p(y_k|y_{1:k-1})} \\ &\propto p(y_k|y_{1:k-1}, Q_{0:k})p(Q_{0:k-1}|y_{1:k-1})p(Q_k|Q_{0:k-1}). \end{aligned}$$

By substituting equations (9) and (10) into equation (8), due to the Markov property of the states and conditional independence of measurements, the weight equation can then be shown to be

$$\begin{aligned} w_k^i &\propto \frac{p(y_k|y_{1:k-1}, Q_{0:k}^i)p(Q_{0:k-1}^i|y_{1:k-1})p(Q_k^i|Q_{0:k-1}^i)}{q(Q_k^i|Q_{0:k-1}^i, y_{1:k})q(Q_{0:k-1}^i|y_{1:k-1})} \\ &\propto w_{k-1}^i \frac{p(y_k|Q_k^i)p(Q_k^i|Q_{k-1}^i)}{q(Q_k^i|Q_{0:k-1}^i, y_{1:k})}. \end{aligned}$$

The importance density can be given as the $q(Q_k^i|Q_{0:k-1}^i, y_{1:k}) = q(Q_k^i|Q_{k-1}^i, y_k)$ and it is easier to draw samples and perform subsequent importance weight calculations with transition prior $q(Q_k^i|Q_{0:k-1}^i, y_{1:k}) = p(Q_k^i|Q_{k-1}^i)$. A filtered estimate of $p(Q_k|y_{1:k})$ can be derived by integrating equation (7) as below,

$$p(Q_k|y_{1:k}) \approx \sum_{i=1}^{N} w_k^i \delta(Q_k - Q_k^i). \tag{10}$$

It can be shown that as $N \longrightarrow \infty$, the approximation equation (10) approaches the true density $p(Q_k|y_{1:k})$. The set of particles is updated and reweighted using a recursive version of importance sampling. An additional resampling procedure such as Systematic resampling is used when the effective number of particles $N_{eff} = \frac{1}{\sum_{i=1}^{N}(w_k^i)^2}$ is significantly less than the total number of particles for removing particles with very small weights and duplicating particles with large weights (Arulampalam, 2003).

The General Scheme of the SMC Filter

For $\quad i = 1, \ldots, N$

- Draw $Q_k^i \sim p(Q_k|Q_{k-1}^i, y_k)$
- Assign the particle a weight, w_k^i, according to equation (10) and normalize them to sum to unity.
- Calculate the effective number of particles, if $N_{eff} < N$, perform resampling.

4 Bayesian Estimation of the SNSS-RC Parameters

Let $\mathbf{\Theta} = (\mu_r, \sigma_r^2, \lambda_r, C, R)$ be a full parameter vector of the entire class of SNSS-RC model with considering constatnt value of V and α. Knowing the exact value of parameter is unrealistic assumption, thus a Bayesian approach to the model of equation (6) consider a random variable vector with a prior distribution as a degree of belief in different values of this parameter vector. Choosing a suitable prior density $\pi(\mathbf{\Theta})$ for $\mathbf{\Theta}$ and computing the joint posterior density of

$$p(Q_{1:n}, \mathbf{\Theta}|y_{1:n}) = p(\mathbf{\Theta}|y_{1:n})p(Q_{1:n}|\mathbf{\Theta}, y_{1:n}), \tag{11}$$

in the off-line case is a main purpose of Bayesian perspective by using PMMH scheme which can be considered as an exact approximation of a "Marginal Metropolis–Hastings" (MMH) update targeting directly the marginal density $p(\boldsymbol{\Theta}|y_{1:n})$ of $p(Q_{1:n}, \boldsymbol{\Theta}|y_{1:n})$ (Andrieu et al., 2010). This marginal posterior density is proportional to the product of the prior density of parameters and the likelihood function

$$p(\boldsymbol{\Theta}|y_{1:n}) \propto \pi(\boldsymbol{\Theta}) L_{\boldsymbol{\Theta}}(y_{1:n}),$$

where the likelihood function $L_{\Theta}(y_{1:n})$ can be written as bellow,

$$L_{\boldsymbol{\Theta}}(y_{1:n}) = \prod_{k=1}^{n} p_{\boldsymbol{\Theta}}(y_k|y_{1:k-1}), \tag{12}$$

where $p_{\boldsymbol{\Theta}}(y_k|y_{1:k-1})$ in equation (12) can be approximated by using

$$\begin{aligned} p_{\boldsymbol{\Theta}}(y_k|y_{1:k-1}) &= \int p_{\boldsymbol{\Theta}}(y_k|Q_k) p_{\boldsymbol{\Theta}}(Q_k|y_{1:k-1}) dQ_k \\ &= \int p_{\boldsymbol{\Theta}}(y_k|Q_k) p_{\boldsymbol{\Theta}}(Q_k|Q_{k-1}) p_{\boldsymbol{\Theta}}(Q_{k-1}|y_{1:k-1}) dQ_{k-1:k}. \end{aligned}$$

By constructing the Q_k^i's that are distributed according to importance density $q_{\boldsymbol{\Theta}}(Q_k|Q_{0:k-1}^i, y_{1:k})$ and corresponding importance weights

$$\alpha_k(Q_{k-1:k}) = \frac{p_{\boldsymbol{\Theta}}(y_k|Q_k) p_{\boldsymbol{\Theta}}(Q_k|Q_{k-1})}{q_{\boldsymbol{\Theta}}(Q_k|Q_{0:k-1}, y_{1:k})},$$

the Monte Carlo approximation can be written

$$p_{\boldsymbol{\Theta}}(y_k|y_{1:k-1}) \approx \frac{1}{N} \sum_{i=1}^{N} \alpha_k^i(Q_{k-1:k}),$$

thus, the approximation of likelihood function can be derived as,

$$\hat{L}_{\boldsymbol{\Theta}}(y_{1:n}) = \prod_{k=1}^{n} \frac{1}{N} \sum_{i=1}^{N} \alpha_k^i(Q_{k-1:k}).$$

The distributions of common parameters as prior distributions are set as: $\mu_r \sim N(\bar{\mu}_r, \sigma_{\mu_r}^2)$, $(\sigma_r^2)^{-1} \sim IG(\alpha, \beta)$, $\lambda_r \sim N(\bar{\lambda}_r, \sigma_{\lambda_r}^2)$, $C \sim N_+(0, \theta)$ and $R \sim$

$N(\bar{R}, \sigma_R^2)$, where $IG(\alpha, \beta)$, $N_+(0, \theta)$ denote the inverse gamma with mean $\frac{\beta}{\alpha-1}$ and half normal distribution, respectively. Without loss of generality the independence of parameters is considered, thus the prior distribution of $\mathbf{\Theta}$ can be written as,

$$\pi(\mathbf{\Theta}) \quad = \quad \pi(\mu_r)\pi(\sigma_r^2)\pi(\lambda_r)\pi(C)\pi(R).$$

Sampling from $p(Q_{1:n}|\mathbf{\Theta}, y_{1:n})$ defined in equation (11) can be done when the whole particles and weights $\{Q_k^i, w_k^i\}, k = 1, \ldots, n, \quad i = 1, \ldots, N$ are obtained by filtering procedure (Andrieu et al., 2010).

The General Scheme of Sampling from $p(Q_{1:n}|\Theta, y_{1:n})$

(i) Choose $\hat{Q}_n = Q_n^i$ with probability $\frac{w_n^i}{\sum_{i=1}^N w_n^i}$.

(ii) For $k = n-1, \ldots, 1$

- Calculate $w_{k|k+1}^i \propto w_k^i p_{\mathbf{\Theta}}(\hat{Q}_{k+1}|Q_k^i)$ for $i = 1, \ldots, N$.
- Choose $\hat{Q}_k = Q_k^i$ with probability $\frac{w_{k|k+1}^i}{\sum_{i=1}^N w_{k|k+1}^i}$.

$\hat{Q}_{1:n} = (\hat{Q}_1, \ldots, \hat{Q}_n)$ is an approximate realisation of $p(Q_{1:n}|y_{1:n}, \mathbf{\Theta})$.
The following form of proposal density q for a Metropolis–Hastings (MH) update is suggested

$$q\{(\tilde{\mathbf{\Theta}}, \tilde{Q}_{1:n})|(\mathbf{\Theta}, Q_{1:n})\} = q(\tilde{\mathbf{\Theta}}|\mathbf{\Theta})p(\tilde{Q}_{1:n}|y_{1:n}, \tilde{\mathbf{\Theta}}).$$

The resulting MH acceptance ratio is given by

$$\frac{p((\tilde{\mathbf{\Theta}}, \tilde{Q}_{1:n})|y_{1:n})q\{(\mathbf{\Theta}, Q_{1:n})|(\tilde{\mathbf{\Theta}}, \tilde{Q}_{1:n})\}}{p((\mathbf{\Theta}, Q_{1:n})|y_{1:n})q\{(\tilde{\mathbf{\Theta}}, \tilde{Q}_{1:n})|(\mathbf{\Theta}, Q_{1:n})\}} = \frac{L_{\tilde{\mathbf{\Theta}}}(y_{1:n})\pi(\tilde{\mathbf{\Theta}})q(\mathbf{\Theta}|\tilde{\mathbf{\Theta}})}{L_{\mathbf{\Theta}}(y_{1:n})\pi(\mathbf{\Theta})q(\tilde{\mathbf{\Theta}}|\mathbf{\Theta})}.$$

A schematic overview of the proposed PMMH algorithm is as below.

The Generic Sampling Scheme of PMMH

(i) At iteration $k = 0$,

- Set $\mathbf{\Theta}^{(0)}$ arbitrarily.

- Run a SMC algorithm given $\mathbf{\Theta}^{(0)}$ targeting $p(Q_{1:n}|y_{1:n}, \mathbf{\Theta})$, sample $Q_{1:n}^{(0)} \sim \hat{p}(Q_{1:n}|y_{1:n}, \mathbf{\Theta}^{(0)})$, and compute the marginal likelihood estimate $\hat{L}_{\mathbf{\Theta}^{(0)}}(y_{1:n})$.

(ii) At iteration $k \geqslant 1$,

- Sample $\tilde{\mathbf{\Theta}} \sim q(\mathbf{\Theta}|\mathbf{\Theta}^{(k-1)})$.
- Run an SMC algorithm given $\tilde{\mathbf{\Theta}}$ targeting $p(Q_{1:n}|y_{1:n}, \mathbf{\Theta})$, sample $\tilde{Q}_{1:n} \sim \hat{p}(Q_{1:n}|y_{1:n}, \tilde{\mathbf{\Theta}})$, and compute the marginal likelihood estimate $\hat{L}_{\tilde{\mathbf{\Theta}}}(y_{1:n})$.
- With probability

$$\min\left\{1, \frac{L_{\tilde{\mathbf{\Theta}}}(y_{1:n})\pi(\tilde{\mathbf{\Theta}})q(\mathbf{\Theta}|\tilde{\mathbf{\Theta}})}{L_{\mathbf{\Theta}}(y_{1:n})\pi(\mathbf{\Theta})q(\tilde{\mathbf{\Theta}}|\mathbf{\Theta})}\right\}.$$

Set $\mathbf{\Theta}^{(k)} = \tilde{\mathbf{\Theta}}$, $Q_{1:n}^{(k)} = \tilde{Q}_{1:n}$ and $\hat{L}_{\mathbf{\Theta}^{(k)}}(y_{1:n}) = \hat{L}_{\tilde{\mathbf{\Theta}}}(y_{1:n})$; otherwise set $\mathbf{\Theta}^{(k)} = \mathbf{\Theta}^{(k-1)}$, $Q_{1:n}^{(k)} = Q_{1:n}^{(k-1)}$ and $\hat{L}_{\mathbf{\Theta}^{(k)}}(y_{1:n}) = \hat{L}_{\mathbf{\Theta}^{(k-1)}}(y_{1:n})$.

The Normal distribution for R, λ_r, μ_r and lognormal for σ_r^2, C are considered as the proposal density.

5 Simulation Study

In this section, some simulation results written in R software are presented to show the performance of the proposed methods. For this purpose, the simulation is carried out for different choices of μ, σ as the mean and variance of colored noise, voltage (V), capacitor (C), resistance (R), $\mu_r, \sigma_r^2, \lambda_r$ as the mean, variance and shape parameter of skew normal measurement noise with considering the intensity of noise ($\alpha = 0.1$) in the SNSS-RC model that is shown in Table 1. In this Table $\mathbf{\Theta} = (C, R, \mu_r, \sigma_r^2, \lambda_r)$ is considered as a unknown parameter vector and μ, σ, V as the known constant values. The simulated charge, Q_t, as the hidden state of the system and the observations, y_t, from data set 3 are shown in Figure 1.
The true states for the exemplar run of data set 1 in the top panel and the estimated states via SMC filter with colored noise in the middle panel and comparison of these paths are shown in the bottom panel of Figure 2 with considering $\Delta t = 0.01$ and $N = 1000$ particles. We can see that the mean of

Table 1. Simulation from four different data sets.

Noise of Dynamic Model	Data Set	Constant μ	σ	V	Unknown Parameter C	R	μ_r	σ_r^2	λ_r
Colored	1	2	2	10	4	12	0	2	2
	2	2	2	10	7	14	2	2	6
White	3	-	-	15	15	5	1	1	8
	4	-	-	12	3	6	2	1	9

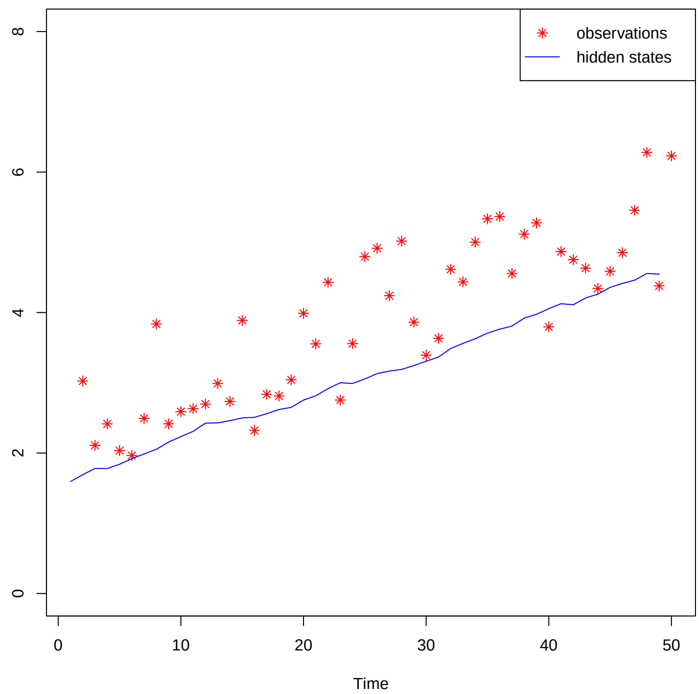

Figure 1. The simulated Charge Q_t (solid line) and the observations y_t (star) from data set 3.

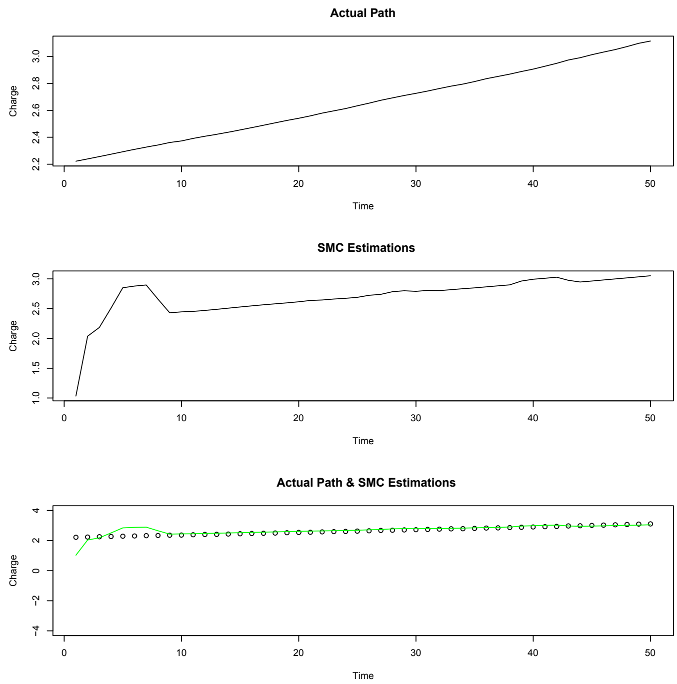

Figure 2. The true value of the charge at time k in the top panel, the estimated charge via SMC filter and comparison with true value in the middle and bottom panel from data set 1.

the SMC filter is close to the true states. $k = 0, \ldots, 10$ values of y_k, Q_k, e_k as the observation, the true charge and the estimated charge respectively from data sets 1 and 4 with colored and white noise as dynamic noise of system are shown in Table 2 with considering $\Delta t = 0.02$ and $N = 1000$ particles. The simulation results show that the estimated charge is close to the true value with different dynamic noises. The coverage percentage of the normal and the skew normal as the measurement noise when the data is simulated from data sets 2 and 3 with different values of λ_r and n is shown in Table 3 with considering $T = 1$. This quantity is computed as the percent of the relative frequency for the hidden states that are in the confidence interval. As we see, the coverage percentage of the skew normal is more than the one of the normal when the data is simulated from the skew normal as the measurement noise. Estimation of the unknown parameters is investigated based on Bayesian inference. This method has been tested by using PMMH

Table 2. The values of the observations, the true charges and the estimated charges.

Data Set	k	0	1	2	3	4	5	6	7	8	9	10
	y_k	-	3.4821	3.5072	4.5859	3.9071	4.7610	4.7525	0.6895	2.0364	4.0669	1.5685
1	Q_k	2.2222	2.2392	2.2563	2.2752	2.2928	2.3094	2.3274	2.3422	2.3615	2.3731	2.3916
	e_k	1.0319	2.0378	2.1835	2.5105	2.8507	2.8788	2.8966	2.6595	2.4304	2.4466	2.4547
	y_k	-	2.5757	3.2674	2.1754	2.6406	3.6709	2.2096	3.7490	3.1855	4.1384	2.7059
4	Q_k	-0.0299	0.0091	0.0532	0.0914	0.1295	0.1719	0.2147	0.2590	0.2970	0.3354	0.3736
	e_k	0.9654	0.2221	0.3607	0.0728	0.1126	0.1525	0.1922	0.2320	0.2718	0.3114	0.3510

Table 3. The coverage percentage of Q_k for (Normal, Skew Normal) as measurement noise for diffrent value of λ_r

Data Set		$\lambda_r = 1$	$\lambda_r = 5$	$\lambda_r = 10$
	$n = 20$	(0.3684,0.7368)	(0.0526,0.8947)	(0.0010,0.5789)
2	$n = 50$	(0.2040,0.7346)	(0.1632,0.8368)	(0.1020,0.7959)
	$n = 100$	(0.0100,0.6464)	(0.0303,0.5155)	(0.0101,0.5656)
	$n = 20$	(0.6842,0.7894)	(0.1578,0.8947)	(0.3684,0.8421)
3	$n = 50$	(0.2040, 0.7346)	(0.2448,0.8367)	(0.1836, 0.8775)
	$n = 100$	(0.0707,0.8585)	(0.0707,0.8181)	(0.0101, 0.7979)

Table 4. The numerical values of estimators $\hat{C}$, $\hat{R}$, $\hat{\mu_r}$, $\hat{\sigma}_r^2$ and $\hat{\lambda_r}$ with different values of Δt, the data in parenthesis are Mean square errors.

	Data Set	Bayesian Estimator (Mean Square Error)				
		$\hat{C}$	$\hat{R}$	$\hat{\mu}_r$	$\hat{\sigma}_r^2$	$\hat{\lambda}_r$
	1	3.9709(0.0469)	11.9534(0.0529)	0.0448(0.0678)	1.9877(0.0412)	1.9779(0.0608)
$\Delta t = 0.1$	2	7.0305(0.0331)	13.9800(0.0768)	1.9551(0.0663)	2.0251(0.0316)	5.9793(0.0479)
	3	14.9365(0.0748)	4.9133(0.1077)	1.1232(0.1396)	1.0304(0.0447)	8.0333(0.0387)
	4	3.0063(0.0574)	5.8466(0.2135)	1.8937(0.1095)	1.0295(0.0489)	8.9123(0.1667)
	1	4.0908(0.1568)	11.8226(0.2487)	0.0110(0.1113)	1.9081(0.1618)	2.1423(0.2414)
$\Delta t = 0.01$	2	7.0467(0.1170)	14.1739(0.1800)	2.1428(0.1691)	2.0310(0.1303)	6.3005(0.4374)
	3	15.0356(0.1717)	4.9693(0.2135)	0.8592(0.2251)	0.9954(0.1697)	7.8417(0.1913)
	4	2.8788(0.2068)	6.1123(0.1786)	1.8984(0.1584)	1.994(0.3024)	6.1123(0.1786)

scheme on the mentioned four simulated data sets with different time step Δt. For this scheme $N = 1000$ particles are used. The results of estimation and relative mean square error (MSE) of estimators from this method are shown in Table 4. It can be claimed that the estimated results are very close to the exact values of parameters based on MSE. In Figures 3 and 4, the evolution of the Metropolis-Hastings chains over 5000 iterations for the unknown parameters of data set 2 and 4 with considering $\Delta t = 0.01$ and 0.1 is illustrated. According to these Figures, the Metropolis-Hastings algorithm seems to converge and the posterior means of parameters as Bayesian estimations are obtained by using 1000 last reordered iterations. We find that the Bayesian estimations of $C, R, \mu_r, \sigma_r^2, \lambda_r$ are equal to 7.0325, 13.9656, 1.9663, 2.0047, 5.8695 in Figure 3 and are equal to 2.9852, 6.2136, 1.9865, 1.9636, 6.1412 in Figure 4.

6 Conclusion

In this paper, we proposed a state space model for stochastic RC electrical circuit by considering the SDE of this circuit as the dynamic model of state space system with white and colored process noise and the skew normal measurement noise . The assumption of skew normal measurement noise in the model makes variation (non-Gaussian) to the modeling of this circuit in comparison with the work (RL electrical circuit) which is considered in (Farnoosh and Hajrajabi, 2013) and makes the model be applicable in various types of real time projects. The SMC filter is proposed to track the hidden charge from the noisy observation. The results show that by this scheme the estimated value of the charge is very close to that of the exact value. Furthermore, it is shown that the coverage percentage of skew normal is more than the one of normal when the data is simulated from the skew normal as the measurement noise. It is recommended to apply the Bayesian approach using the PMMH scheme for estimation of the unknown parameters. Results show that the estimation of parameters based on this approach are achieved after low iterations.

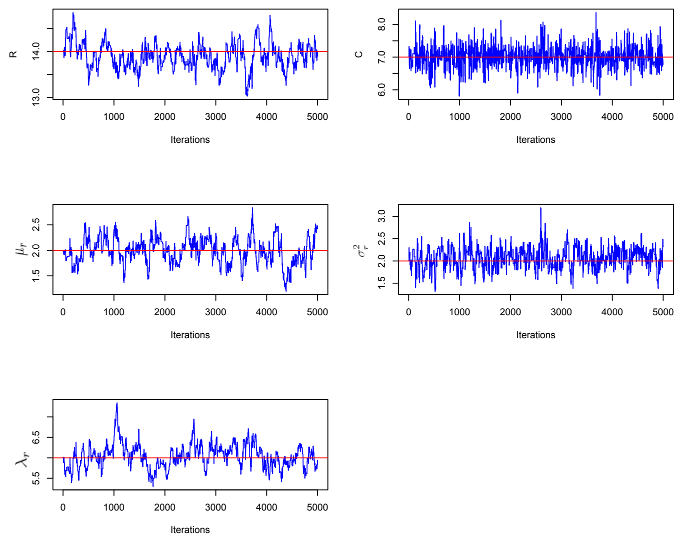

Figure 3. Evolution of the Metropolis-Hastings chains over 5000 iterations for the unknown parameters in data set 2 with considering $\Delta t = 0.01$.

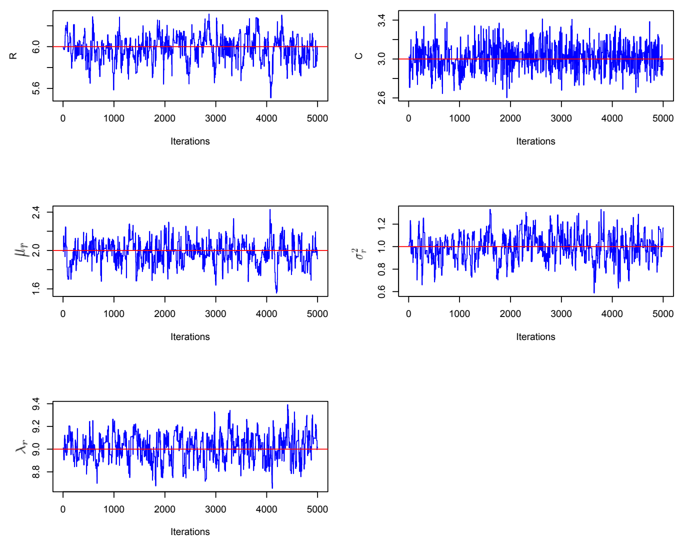

Figure 4. Evolution of the Metropolis-Hastings chains over 5000 iterations for the unknown parameters in data set 4 with considering $\Delta t = 0.1$.

References

Anderson, B.D.O. and Vongpanitlerd, S. (1973). *Network Analysis and Synthesis, Hall, Englewood Cliffs, New Jersey.*

*Andrieu, C., Doucet, A., Singh, S.S. and Tadic, V. (2004). Particle Methods for Change Detection, Identification and Control, Proceedings of the IEEE, **92**, 423-438.*

*Andrieu, C., Doucet, A. and Holenstein, R. (2010). Particle Markov Chain Monte Carlo Methods, Journal of the Royal Statistical Society: Series B, **72**, 269-342.*

*Arulampalam, M.S., Maskell, S., Gordon, N. and Clapp, T. (2003). A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking, IEEE Transactions on Signal Processing, **50**, 174-188.*

*Azzalini, A. (2005). The Skew-normal Distribution and Related Multivariate Families, Scandinavian Journal of Statistics, **32**, 159-188.*

*Campillo, F. and Rossi, V. (2009). Convolution Particle Filter for Parameter Estimation in General State-Space Models, IEEE Transactions on Aerospace and Electronic Systems, **45**, 1063-1072.*

Coquelin, A., Deguest, R. and Munos, R. (2008). Perturbation Analysis for Parameter Estimation in Continuous Space HMMs, Technical report, INRIA Sequel.

Doucet, A., Godsill, S.J. and Andrieu, C. (2000). On Sequential Monte Carlo Sampling Methods for Bayesian Filtering, *Statistics and Computing*, **10**, 197-208.

Farnoosh, R., Nabati, P., Hajrajabi, A. (2012). Parameters Estimation for RL Electrical Circuits based on Least Square and Bayesian Approach, *The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, **31**, 1711-1725.

Farnoosh, R. and Hajrajabi, A. (2013). Estimation of Parameters in the State Space Model of Stochastic RL Electrical Circuit, *The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, **32**, 1082-1097.

Fearnhead, P. (2002). Markov Chain Monte Carlo, Sufficient Statistics and Particle Filter, *Journal of Computational and Graphical Statistics*, **11**, 848-862.

Julier, S.J. and Uhlmann, J.K. (2004). Unscented Filtering and Nonlinear Estimation, *Proceedings of the IEEE*, **92**, 401-422.

Kantas, N., Doucet, A., Singh, S.S. and MacIejowski, J.M. (2009). An Overview of Sequential Monte Carlo Methods for Parameter Estimation in General State-space Models, IFAC Proceedings Volumes (IFAC-PapersOnline) 15 (PART 1), france, 774-785.

Kitagawa, G. (1998). Self-organizing Time Series Model, *Journal of the American Statistical Association*, **93**, 1203-1215.

Liu, J.S. (2001). *Monte Carlo Strategies in Scientific Computing,* Springer.

Oksendal, B. (2000). *Stochastic Differential Equations: An Introduction with Applications,* Springer, Berlin.

Poyiadjis, G., Doucet, A. and Singh, S.S. (2009). Sequential Monte Carlo for Computing the Score and Observed Information Matrix in State-space Models with Applications to Parameter Estimation, Technical report CUED/F-INFENG/TR.628, Cambridge University.

Poyiadjis, G., Doucet, A. and Singh, S.S. (2011). Particle Approximations of the Score and Observed Information Matrix in State Space Models with Application to Parameter Estimation, *Biometrika*, **98**, 65-80.

Sarkka, S. (2007). On Unscented Kalman Filtering for State Estimation of Continuous-time Nonlinear Systems, *The IEEE Transactions on Automatic Control*, **52**, 1631-1641.

Vats, V.B. and Parthasarathy, H. (2008). Parameter Estimation for Time Varying Nonlinear Circuit from State Analysis and Simulation, *Communications in Nonlinear Science and Numerical Simulation*, **13**, 839-846.

Vongpanitlerd, S. (1970). Reciprocal Lossless Synthesis Via State-variable Techniques, *IEEE Transactions on Circuit Theory*, **17**, 630-632.

Wills, A., Schon, T. and Ninness, B. (2008). Parameter Dstimation for Discrete Time Nonlinear Systems Using EM, Proceedings of the 17th IFAC World Congress, Seoul, Korea.

Wu, Y., Hu, D., Wu, M. and Hu, X. (2006). A Numerical-integration Perspective on Gaussian Filters, *IEEE Transactions on Signal Processing*, **54**, 2910-2921.

Yarlagadda, R. (1972). Network Synthesis - a State-space approach, *IEEE Transactions on Circuit Theory*, **19**, 227-232.

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