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Constrained Optimal Design of $\overline{X}$ Control Chart for Correlated Data under Weibull Shock Model with Multiple Assignable Causes and Taguchi Loss Function

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Abstract. A proper method of monitoring a stochastic system is to utilize the control charts of statistical process control in which a drift in characteristics of output may be due to one or several assignable causes. In the establishment of $\overline{X}$ charts, an assumption is made that there is no correlation within the samples. However, in practice, there are many industrial cases in which the correlation does exist within the samples. It would be more appropriate to assume that each sample is a realization of a multivariate normal random vector. Although some research works have been done on the economic design of control charts with single assignable cause with correlated data, the economic statistical design of $\overline{X}$ control chart for correlated data under Weibull shock model with modified Taguchi loss function have not been presented yet. Using modified Taguchi loss function in the concept of quality control charts with economic and economic statistical design leads to better decisions in the industry. Based on the optimization of the average cost per unit of time and different combination values of Weibull distribution parameters, optimal design values of sample size, sampling interval and control limit coefficient were derived and calculated. Then the cost mod-

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els under non-uniform and uniform sampling scheme were compared. The results revealed that the model under multiple assignable causes with correlated samples with non-uniform sampling has a lower cost than that with uniform sampling.

Keywords. Economic statistical design; $\overline{X}$ control chart; multiple assignable causes; Weibull shock model; correlated data; Taguchi loss function.

MSC 2010: 62P30.

1 Introduction

The need for perfection and elimination of products that do not conform to specification was the main reason and motivation behind coming up with control charts. The method of SPC firstly started from Shewhart (1924) control charts. Control charts can be used for monitoring the production process and eliminating the effect of one or several assignable causes. Various control chart techniques have been developed and widely applied in industries. Saniga and Shirland (1977) showed that the averages control chart (or the $\overline{X}$ chart) dominates the use of any other control chart techniques if the quality is measured on a continuous scale. In designing any control chart, three fundamental questions need to be answered about so-called tunic design parameters. First, what should be the sample size? Second, how often the samples should be taken? Third, what should be the control limits coefficient? The optimal choice of chart parameters has a huge impact on the performance of a control chart.

In the existing literature, different methods have been developed for optimizing the design parameters. The first method was the heuristic methods in which some of the quality control Gurus suggested different values for the parameters. Then statistical methods were used to calculate the optimal values of the design parameters. The sample size and control limits can be determined based on the Type I error probability and pre-specified power of the test at the desired levels in statistical design. Girshick and Rubin (1952) presented the concept of economic design for the first time and their study became a basis for subsequent research. Duncan (1956) in his paper adopted the economic design of $\overline{X}$ control charts under exponential shock model. Although the economic consequences are considered in the economic design, but this design is very poor in terms of statistical criteria (Woodall, 1986). To remove the weaknesses of the economic model, Saniga (1989) proposed a new model to combine the benefits of both pure statistical and economic designs while minimizing their weaknesses. This leads to propose the economic statistical design, where statistical constraints are incorporated into the economic design. The economic statistical design can be viewed as a cost improvement approach to statistical designs, and as a statistical performance improvement approach to economic designs. Thus, it is one of the most versatile approaches in control chart designs, as it considers both cost and statistical performance of the control charts.

In Duncan's paper (1956), only single assignable cause made a shift in process mean. In industry, there are some situations when multiple assignable causes affected the model. Therefore, many researchers are interested in presenting a model in these situations (Gibra, 1981; Chung, 1991; Yang et al., 2010; Nenes et al., 2015; Ghanaatiyan et al., 2016; Tasias and Nenes, 2016; Salmasnia et al., 2017).

Duncan (1971) extended his model from single assignable cause to multiple assignable causes where the assignable causes occurred independently. Based on Duncan's model, Yu et al. (2010) presented an economic statistical design for control chart with multiple assignable causes and imposed constraints on Type I and Type II errors.

Economic and economic statistical design of control charts need to have a probability distribution for a process failure mechanism to put process costs in one model. Transition in the process from the state of control to out of control called process failure mechanism (PFM) or shock model. Many distributions such as Exponential, Weibull, Generalized exponential, Burr XII, Gamma, Pareto and ..., are used as a failure mechanism (Duncan, 1956; Banerjee and Rahim, 1988; Moghadam et al., 2016; Heydari, et al., 2016; Al-Oraini and Rahim, 2002; Kraleti and Kambagowni, 2010). Since using the distributions with increasing failure rate corresponds to reality in the industry, Banerjee and Rahim (1988) used the Weibull distribution instead of exponential distribution to generalize Duncan (1956) model under non-uniform sampling scheme. Weibull distribution can be used to simulate various situations by varying its scale and shape parameters. Based on the cost model of Banerjee and Rahim (1988) and Duncan (1971) model, Chen and Yang (2002) presented the economic design of control charts under Weibull shock model with multiple assignable causes and variable sampling intervals.

The measurements within the samples in the above-mentioned models are totally assumed to be independently distributed in the design procedure of a control chart. Leavenworth and Grant (2000) stated that this assumption may not be defendable in some specific processes; for example, the collected measurements within a sample from the production process, which comprises multiple but similar characteristics in a single part. Other specific examples include several cavities on a single casting, multiple pins on an integrated circuit chip, or multiple contact pads on a single machine mount, which may be correlated. Neuhardt (1987) investigated the effects of correlation existing within a subgroup in a control chart. Yang and Hancock (1990) extended Neuhardt's work to determine the effect of correlated data on $\overline{X}$, R, S and S^2 charts by Monte Carlo Simulation studies. Based on the results of their studies, they proposed that if a positive correlation exists but is not recognized, a significantly larger Type I error probability than the assumed one will be generated in an control chart. Chou et al. (2001) combined the Yang and Hancock's (1990) model with the economic design approach to determine the parameters of average control charts under correlated samples. Liu et al. (2002) employed Yang and Hancock's correlation model and the fixed-sampling-interval (FSI) policy to develop a minimum-loss design of charts for correlated data. Chen and Yeh (2010) combined Banerjee and Rahim's (1988) cost model with Yang and Hancock's (1990) correlation model to develop an economic statistical design model of $\overline{X}$ charts for processes with correlated data and the Gamma failure mechanism.

The use of control charts implies that quality loss is considered as the cost when the quality characteristics are outside the specification limits. All products falling within the control limits are considered as having the same quality regardless of the deviation of their quality characteristic from its target value. However, this is not the case when it comes to real life examples in which any deviation from the target value will incur a cost to the customers. Taguchi et al. (1989) defined quality loss as “the loss to society caused by the product after it is shipped out”. His quadratic loss function is well known and has been widely used in all fields. It is used to estimate the quality loss of a product when its quality characteristic deviates from its target value. Until now, a lot of economic and economic-statistical design developed for control chart by the combination of classic models like Duncan and Lorenzen Vance model by Taguchi loss function (Safaei et al., 2012; Al-Ghazi et al., 2007; Yang, 1998; Koo and Lin, 1992). In economic and economic- statistical design, loss cost under control and out of control is calculated with Taguchi loss function in many research works (Serel and Moskowitz, 2008; Elsayed and Chen 1994). Yu and Chen (2009) presented the economic-statistical design of $\overline{X}$ control chart with Taguchi loss function under multiple assignable causes.

The economic statistical design of $\overline{X}$ control chart under Weibull shock model for correlated data with multiple assignable causes and Taguchi loss function is not presented yet and we in this paper present economic statistical design of $\overline{X}$ control chart under Weibull shock model with multiple assignable causes and Taguchi loss function for correlated data by using the concepts of $AATS$ (Average time between occurrence shifts in the process mean owing to assignable causes and receiving the right alarm from control chart) and ANF (Average numbers of false alarm in the quality cycle). In our paper, by considering fixed sampling interval (uniform sampling scheme), we calculate the average cost for the cycle and compare our findings with the average cost in the case of non-uniform sampling. To calculate cost functions for uniform and non-uniform sampling schemes, we presented and proved the formulas of based on multiple assignable causes and in the case of uniform and non-uniform sampling schemes. To construct economic statistical design we used penalty approach and both the statistical properties and optimization of loss cost have been considered simultaneously. According to literature review, Table 1 summarizes the characteristics of the existing researches in the literature and shows the position of our paper.

The structure of this paper is as follows. In Section 2, a correlated model overview is given. In Section 3, a cost model offered with multiple assignable causes with non-uniform and uniform sampling schemes. In this section integrated cost model with modified Taguchi loss function also was presented. Section 4 includes the economic statistical design. Section 5 includes a real industrial example and the way of determining input parameters and optimizing cost model based on this input parameters by considering economic and economic statistical designs. The comparison between cost model under multiple assignable causes with uniform and non-uniform schemes also presented in Section 5. Section 6 includes of sensitivity analysis and investigation of the amount of change in loss cost and design parameters by the variation of input parameters. Finally, the conclusion is presented in Section 7.

Table 1. Summarized literature review

Papers	Assignable Cause	PFM	Design Type	Samples Relation	Objective	Integrated with Taguchi Loss Function
Duncan (1956)	Single	Exponential	Economic	Uncorrelated	Cost	No
Duncan (1971)	Multiple	Exponential	Economic	Uncorrelated	Cost	No
Lorenzen and Vance (1986)	Single	Exponential	Economic	Uncorrelated	Cost	No
Neuhardt (1987)	Single	-	Statistical	Correlated	α and $1-\beta$	No
Banerjee and Rahim (1988)	Single	Weibull	Economic	Uncorrelated	Cost	No
Yang and Hancock (1990)	Single	-	Statistical	Correlated	α and $1-\beta$	No
Rahim and Banerjee (1993)	Single	Increasing hazard rate distribution	Economic	Uncorrelated	Cost	No
Elsayed, and Chen (1994)	Single	Exponential	Economic	Uncorrelated	Cost	Yes
Zhang and Berardi (1997)	Single	Weibull	Economic- Statistical	Uncorrelated	Cost and α and $1-\beta$	No
Chen and Yang (2002)	Multiple	Weibull	Economic	Uncorrelated	Cost	No
Ben-Daya, and Duffuaa (2003)	Single	Exponential	Economic	Uncorrelated	Cost	Yes
Aloraini and Rahim (2002)	Single	Gamma	Economic- Statistical	Uncorrelated	Cost and α and $1-\beta$	No
Chen et al. (2007)	Single	Exponential	Economic	Correlated	Cost	No
Yu and Chen (2009)	Multiple	Exponential	Economic- Statistical	Uncorrelated	Cost and α and $1-\beta$	Yes
Chen and Yeh (2010)	Single	Gamma	Economic- Statistical	Correlated	Cost and α and $1-\beta$	No
Kraleti and Kambagowni (2010)	Single	Pareto	Economic	Uncorrelated	Cost	No
Yu et al. (2010)	Multiple	Exponential	Economic- Statistical	Uncorrelated	Cost and α and $1-\beta$	No
Safaei et al. (2012)	Single	Exponential	Multiple Objective ESD	Uncorrelated	Cost and α and $1-\beta$	Yes
Heydari et al. (2016)	Single	Burr Xll	Economic- Statistical	Uncorrelated	Cost and α and $1-\beta$	No
Moghadam et al. (2016)	Single	Generalized Exponential	Economic	Uncorrelated	Cost	No
This paper	Multiple	Weibull	Economic- statistical	Correlated	Cost AATS and ANF	Yes

2 Correlated Model Overview

Suppose that the output of manufacturing process has one quality characteristic (X) and X has a normal distribution. In this paper, it is assumed that the time of being under control until ith assignable cause occurs follows a Weibull distribution with below probability density function and increasing hazard rate:

$$f_i(t) = \lambda_i k t^{k-1} \exp(-\lambda_i t^k), \qquad t > 0,\ k \geqslant 1,\ \lambda_i > 0,\ i = 1, 2, \ldots, s. \quad (1)$$

$$r_i(t) = \lambda_i k t^{k-1}, \quad (2)$$

where k is shape parameter and λ_i is scale parameter. Note that when $k = 1, f_i(t)$ becomes exponential distribution with $r_i(t) = \lambda_i$.

The process is monitored by taking samples of size n from X at time intervals $h_1,\ h_1 + h_2,\ h_1 + h_2 + h_3$ and so on. In this case is jth sampling interval and we have $h_1 \geqslant h_2 \geqslant h_3, \ldots$ (The proof of above inequality is given in Appendix B).

Yang and Hancock (1990) assume that each subgroup (samples of size n from X in sampling intervals) is a realization of the random vector, $X = X_1, X_2, X_3, \ldots, X_n$, which has the multivariate normal distribution $N(\mu, V)$, where μ is the vector of mean values and $V = V_{ij},\ i, j = 1, 2, \ldots, n$, is the covariance matrix. In addition, ρ, is the correlation matrix and σ is the process standard deviation. Based on these assumptions, the sample mean can be shown to be normally distributed with mean and variance as follows:

$$E(\overline{X}) = \mu \quad (3)$$

$$V(\overline{X}) = \frac{\sigma^2}{n}[1 + (n-1)\rho] \quad (4)$$

where

$$\rho = \frac{\sum_{i \neq j} r_{ij}}{n(n-1)}. \quad (5)$$

The proof of Equation 4 is given in Appendix C.

Sampling intervals are determined in a way that the probability of shift from a control state is fixed for all intervals when it is in control at the beginning of the interval. In other words, integrated hazard over each interval should be equal.

$$\int_{\omega_j}^{\omega_{j+1}} r_i(t)\,\mathrm{d}t = \int_0^{h_1} r_i(t) \qquad \mathrm{d}t, j = 1, 2, \ldots. \tag{6}$$

Accordingly h_j (for more detail see Appendix D) obtained as follows

$$h_j = [j^{\frac{1}{k}} - (j-1)^{\frac{1}{k}}]h_1, \qquad j = 1, 2, \ldots \tag{7}$$

$$\omega_j = \Sigma_{i=1}^{j} h_i. \tag{8}$$

We need to define the following terms to calculate the average cost per unit of time.
1. p_{ij} is the conditional probability that ith assignable cause $(i = 1, 2, \ldots, s)$ will occur during jth sampling interval $(j = 1, 2, \ldots)$, given that ith assignable cause not occur at time ω_{j-1}.

$$\begin{aligned} p_{ij} &= \frac{\int_{\omega_{j-1}}^{\omega_j} f_i(t)\,\mathrm{d}t}{\int_{\omega_{j-1}}^{\infty} f_i(t)\,\mathrm{d}t} \\ &= \frac{\exp(-\lambda_i \omega_{j-1}^k) - \exp(-\lambda_i \omega_j^k)}{\exp(-\lambda_i \omega_{j-1}^k)} \\ &= 1 - \exp(-\lambda_i(\omega_j^k - \omega_{j-1}^k)). \end{aligned} \tag{9}$$

In Appendix D we show that

$$\omega_j = j^{\frac{1}{k}} h_1$$

hence

$$\begin{aligned} p_{ij} &= 1 - \exp(-\lambda_i(jh_1^k - (j-1)h_1^k)) \\ &= 1 - \exp(-\lambda_i(h_1^k). \end{aligned}$$

Let $p_{ij} = p_i$, for $(i = 1, 2, \ldots, s)$, $(j = 1, 2, \ldots)$.
2. We consider q_{ij} as the unconditional probability that ith assignable cause will occur during jth sampling interval and the process is going to out of

control, so,

$$
\begin{aligned}
q_{ij} &= \int_{\omega_{j-1}}^{\omega_j} f_i(t)\,\mathrm{d}t = e^{-\lambda_i \omega_{j-1}^k} - e^{-\lambda_i \omega_j^k} \\
&= e^{-\lambda_i (j-1) h_1^k} - e^{-\lambda_i j h_1^k} \\
&= (1-p_i)^{j-1} - (1-p_i)^j \\
&= (1-p_i)^{j-1} p_i \qquad (10)\\
q_{i1} &= 1 - e^{-\lambda_i \omega_1^k} = 1 - e^{-\lambda_i h_1^k} \\
&= p_{i1} = p_i
\end{aligned}
$$

3. Suppose that τ_{ij} be the expected time of the in control period within sampling interval h_j, given that ith assignable cause has occurred during this period.

$$
\begin{aligned}
\tau_{ij} &= E(T - \omega_{j-1} \mid \omega_{j-1} < T < \omega_j) \\
&= \frac{\int_{\omega_{j-1}}^{\omega_j} (t-\omega_{j-1}) f_i(t)\,\mathrm{d}t}{q_{ij}}. \qquad (11)
\end{aligned}
$$

So The expected (the time that process be under control) during any one sampling interval is as follows:

$$
\begin{aligned}
\tau_i &= \sum_{j=1}^{\infty} \tau_{ij} q_{ij} = \sum_{j=1}^{\infty} \int_{\omega_{j-1}}^{\omega_j} (t-\omega_{j-1}) f_i(t)\,\mathrm{d}t \\
&= (\frac{1}{\lambda_i})^{\frac{1}{k}} \Gamma(1+\frac{1}{k}) - h_1 p_i (1-p_i) A(1-p_i), \qquad (12)
\end{aligned}
$$

where for $|x| < 1$

$$
A(x) = \sum_{j=0}^{\infty} (j+1)^{\frac{1}{k}} x^j. \qquad (13)
$$

Like all statistical assumptions, it is possible that both Type I and Type II errors occur in control chart. In $\overline{X}$ control chart with correlated data the probability of Type I error is calculated as fallows:

$$\alpha = 2\int_{\frac{L}{\sqrt{1+(n-1)p}}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)\mathrm{d}x. \tag{14}$$

When the ith assignable cause occurs, the process mean shifts. In $\overline{X}$ control chart with correlated data the probability of Type II error is calculated as follows:

$$\beta_i = \int_0^{\frac{L-\delta_i\sqrt{n}}{\sqrt{1+(n-1)p}}} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)\mathrm{d}x - \int_0^{\frac{-L-\delta_i\sqrt{n}}{\sqrt{1+(n-1)p}}} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)\mathrm{d}x. \tag{15}$$

We can see the proof of formulas (14) and (15) in Yang and Hancock (1990).

Let $AATS_i$ be the average time between occurrence shifts in the process mean owing to the i^{th} assignable cause and receiving the right alarm from control chart. Therefore,

$$AATS_i = h_1p_iA(1-p_i) + \frac{\beta_ih_1p_i[p_iA(1-p_i)-(1-\beta_i)A(\beta_i)]}{1-p_i-\beta_i} - (\frac{1}{\lambda_i})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}). \tag{16}$$

The concept of $AATS_i$ is presented well in Figure 1. The proof of above formula is presented in Appendix E.

3 Development of Cost Model

3.1 Model Assumptions

To create a cost model we should consider the following assumptions:

1. The output of the process has a normal distribution with constant mean and variance.
2. When the process is under the control, $\mu = \mu_0$.
3. It is assumed that assignable causes occur based on the Weibull distribution. By assuming that the process begins with the state of control, the time which process be under control has a Weibull distribution.

4. Assignable causes occur independently.
5. Multiple assignable causes produce "step changes" in the process mean from $\mu = \mu_0$ to a $\mu = \mu_i$.
6. In this article, the shift occurred in the process mean is noted by δ_i. Three distributions, uniform, negative exponential and half-normal are considered as a prior for δ_i. Considering these distributions as the prior would cover all values of δ_i in a real industry.
7. The process is not self-correcting. That is, once a transition to an out-of-control state has occurred, the process can be returned to the in-control condition only by management intervention upon appropriate corrective actions.
8. The quality cycle starts with the in-control state and continues until the process is repaired after an out-of-control signal. It is assumed that quality cycle follows a Renewal Reward Process.
9. During the search for an assignable cause, the process is shut down.
10. Each subgroup (samples of size n from X in each sampling interval) is a realization of the random vector, $X = \{X_1, X_2, \ldots, X_n\}$, which has the multivariate normal distribution $N(\mu, V)$, where μ is the vector of mean values and $V = \{V_{ij}\}, i, j = 1, 2, \ldots, n$, is the covariance matrix.

3.2 Cost Function in the Case of Non-uniform Sampling

We assumed that S assignable causes affected the process and the occurrence time of any assignable cause follows Weibull distribution. It is also assumed that after the occurrence of the ith assignable cause, until the discovery of the ith assignable cause, the process will not disturb by any other assignable causes. Thus, if the time until occurrence of assignable causes noted by $T_1^{'}, T_2^{'}, \ldots, T_S^{'}$ then the probability of being in control at time t is:

$$P(T^{'} > t) = P(\min(T_1^{'}, T_2^{'}, \ldots, T_S^{'}) > t) = \exp^{-\lambda_0 t^k}, \qquad (17)$$

where $\lambda_0 = \sum \lambda_i, i = 1, 2, \ldots, s$.

Therefore, the time of being in control until the occurrence of multiple assignable causes follows the Weibull distribution.

$$f_0(t) = \lambda_0 k t^{k-1} \exp(-\lambda_0 t^k) \qquad t > 0,\ \lambda_0 > 0,\ k \geqslant 1. \qquad (18)$$

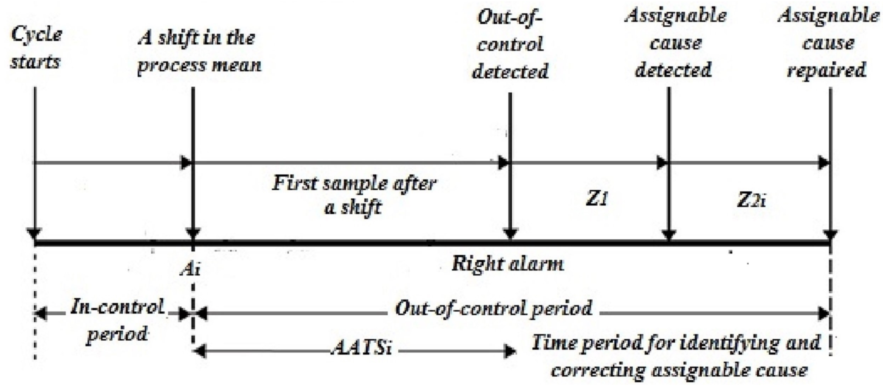

Figure 1. The quality cycle under control and out of control.

We consider P_{0j} as the conditional probability that multiple assignable causes $(i = 1, 2, \ldots, s)$ will occur during jth sampling interval $(j = 1, 2, \ldots)$ given that multiple assignable causes not occur at time ω_{j-1}. We obtain $p_{0j} = 1 - e^{-\lambda_0 h_1^k}$. Here it is assumed $p_{0j} = p_0$.

The average time that the process is in control is:

$$(\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1 + \frac{1}{k}) + Z_0 ANF. \tag{19}$$

In the above formula ANF is the average numbers of false alarm in the quality cycle and is equal to the production of average sample numbers before shift and probability of Type I error (α). First part of the formula (19) is the mean of Weibull distribution. We add second part because of assumption (9) in Section 3 (During the search for an assignable cause, the process is shut down). Therefore we add the total time of finding average numbers of false alarm.

If A is the event of the occurrence of single assignable cause, then average numbers of false alarm calculated as follows:

E (Number of samples are taken before shift)

$$
\begin{aligned}
&= \sum_{j=0}^{\infty} jP(A \in (jh, (j+1)h) && (20)\\
&= \sum_{j=0}^{\infty} j(e^{-\lambda_0 j h_1^k} - e^{-\lambda_0 (j+1) h_1^k})\\
&= \frac{e^{-\lambda_0 h_1^k}}{1 - e^{-\lambda_0 h_1^k}}.
\end{aligned}
$$

Therefore, ANF is equal to:

$$ANF = \alpha \frac{1 - p_0}{p_0}. \quad (21)$$

The average time of cycle is:

$$E(T) = (\frac{1}{\lambda_0})^{\frac{1}{k}} \Gamma(1 + \frac{1}{k}) + Z_0 ANF + AATS + Z_1 + \Sigma(\frac{\lambda_i}{\lambda_0}) Z_{2i}, \quad (22)$$

where

$$AATS = \sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} AATS_i. \quad (23)$$

For a better understanding of $E(T)$ one can see Figure 1.
The average cost of cycle is:

$$
\begin{aligned}
E(C) = D_0(\frac{1}{\lambda_0})^{\frac{1}{k}} \Gamma(1 + \frac{1}{k}) + YANF + \sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} D_{1i} AATS_i + \sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} w_i \\
+ (a + bn) \sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} (\frac{\beta_i}{1 - \beta_i} + \frac{1}{p_0}). \quad (24)
\end{aligned}
$$

For better understanding of $E(C)$ one can see Figure 1. The way of obtaining $\frac{\lambda_i}{\lambda_0}$ in above formula is presented in Appendix F.

In practice, each process starts from in control state. Then because of occurrence of one assignable cause, it goes to out of control state. It is clear that after repairing and fixing the assignable cause, the process returns to the initial state. This cycle is called quality cycle and its model follows the form of a Renewal Reward Process where the average cost per unit time for the cycle $E(A)$ is calculated by the average cost per cycle $E(C)$ divided by the average time per cycle $E(T)$. In economic design, the purpose is optimizing $E(A)$ without any constraint and finding optimal values for sampling interval, sample size and control limits coefficient.

3.3 Cost Function in the Case of Uniform Sampling

To evaluate the relative benefits of non-uniform sampling plan in comparison with uniform sampling plan under multiple assignable causes cost model, and by considering fixed sampling interval, we calculate average time and the average cost for the cycle and analyze them. If h is a fixed sampling interval, then we can obtain $E(T)$ as follows:

$$E(T) = (\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) + Z_0 ANF + AATS + Z_1 + \Sigma(\frac{\lambda_i}{\lambda_0})Z_{2i}, \qquad (25)$$

where

$$AATS = \sum_{i=1}^{s} \frac{\lambda_i}{\lambda_0} AATS_i$$

$$AATS_i = \frac{h}{1-\beta_i} - \tau_i$$

$$\tau_i = (\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) - hQ_i$$

$$Q_i = \sum_{j=1}^{\infty} e^{\lambda_i (jh)^k}$$

$$ANF = \alpha Q, Q = \sum_{j=1}^{\infty} e^{-\lambda_0 (jh)^k}.$$

We also obtain $E(C)$ as fallows:

$$E(C) = D_0(\frac{1}{\lambda_0})^{\frac{1}{k}}\Gamma(1+\frac{1}{k}) + YANF + \sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}D_{1i}AATS_i + \sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}w_i$$
$$+ (a+bn)\sum_{i=1}^{s}\frac{\lambda_i}{\lambda_0}(\frac{1}{1-\beta_i}) + (a+bn)Q. \quad (26)$$

3.4 Improvement of Cost Function by Using Loss Function

To consider the intangible costs, Taguchi loss function is used. In this model, $E(A)$ is used as an economic criterion to assess the measurable costs. To estimate D_0 and D_1 Taguchi loss function is used. Taguchi and et al. (2005) characterized product quality as the loss a product bestows to society from the time the product is delivered and presented the quality loss function as a quality measure. They showed that a quadratic loss function adequately represents economic loss due to the deviation of a quality characteristic from the target value. The Taguchi loss function is presented bellow:

$$L(X) = C(X-t)^2. \quad (27)$$

In the above formula $L(X)$, is the loss connected with the value of quality characteristic X, the target value of the quality characteristic is denoted by t, and C is a constant value depending on the width of the specification and the cost at the specification limits. The customer's desire to have products with high quality are satisfied with above loss function. Ben-Daya and Duffuaa (2003) modified above formula and came up with a nice approximated cost function for the in-control and the out-of-control production assuming the following:

1) The process is monitored using $\overline{X}$ control charts, and it's producing products with symmetric nominal type and bilateral tolerance equal to Δ.

2) During in-control the process is centered at $\mu = \mu_0$ which is the target value, however, during out-of-control, the process mean shifts from μ to $\mu + \delta_i\sigma$.

3) The process is capable; thus the tail of the normal distribution of sample means outside the specification limits can be neglected.

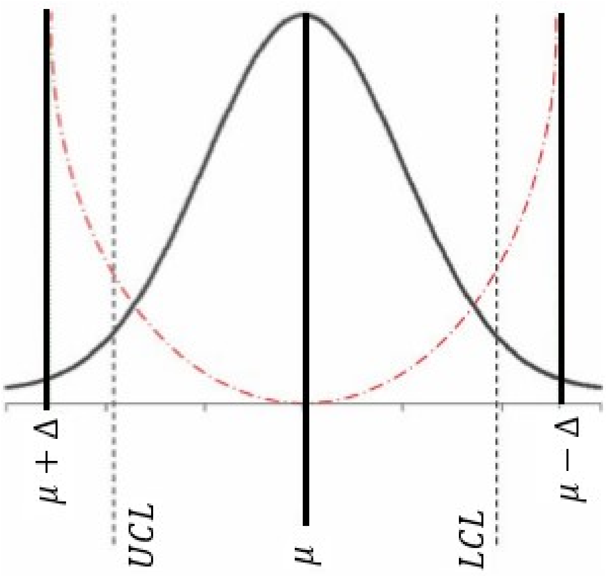

Figure 2. Modified Taguchi loss model.

The in-control and out-of-control costs can be approximately found after examining the Figure 2.

From Figure 2, the approximated in-control cost is given by:

$$LCL = \mu - \frac{L\sigma}{\sqrt{n}}, \qquad UCL = \mu + \frac{L\sigma}{\sqrt{n}} \tag{28}$$

$$L_{in}(n, L) = \frac{A}{\Delta^2} \int_{\mu - \frac{L\sigma}{\sqrt{n}}}^{\mu + \frac{L\sigma}{\sqrt{n}}} (y - \mu)^2 f(y) dy, \tag{29}$$

where y is a random variable denoting sample means of the quality characteristic and $f(y)$ is the normal density function with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

In the same fashion, the approximated out-of-control cost is given by:

$$L_{out,i}(n, L) = \frac{A}{\Delta^2} \int_{\mu - \Delta}^{\mu - \frac{L\sigma}{\sqrt{n}}} (y^{'} - \mu)^2 f(y^{'}) dy^{'} + \frac{A}{\Delta^2} \int_{\mu + \frac{L\sigma}{\sqrt{n}}}^{\mu + \Delta} (y^{'} - \mu)^2 f(y^{'}) dy^{'}, \tag{30}$$

where $y^{'}$ is a random variable denoting sample means of the quality characteristic, and $f(y^{'})$ is the normal density function with mean $\mu \mp \delta\sigma$ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

Using the assumption of neglecting the tails of the normal distribution outside the specification limits, the approximated out-of-control cost can be alternatively expressed as:

$$L_{out,i}(n,L) = \frac{A}{\Delta^2}\int_{-\infty}^{\infty}(y^{'}-\mu)^2 f(y^{'})dy^{'} - \frac{A}{\Delta^2}\int_{\mu-\frac{L\sigma}{\sqrt{n}}}^{\mu+\frac{L\sigma}{\sqrt{n}}}(y^{'}-\mu)^2 f(y^{'})dy^{'}. \quad (31)$$

By doing some algebraic manipulations, both approximated costs can be expressed as:

$$L_{in}(L) = \frac{A\sigma^2}{n\Delta^2}\left[1 - \frac{2L}{\sqrt{2\pi}}e^{-\frac{L^2}{2}} - \alpha\right] \quad (32)$$

$$L_{out,i}(L) = \frac{A\sigma^2}{n\Delta^2}[(1+\delta_i^2 n)(1-\beta_i) + \frac{L+\delta_i\sqrt{n}}{\sqrt{2\pi}}e^{-\frac{(L+\delta_i\sqrt{n})}{2}} + \frac{L-\delta_i\sqrt{n}}{\sqrt{2\pi}}e^{-\frac{(L+\delta_i\sqrt{n})}{2}}. \quad (33)$$

We improve Eq (21) and (23) by using PL_{in} instead of D_0 and $PL_{out,i}$ instead of D_{1i}. Where P is the production rate.

4 Economic-Statistical Design

In statistical design and economic design of control charts, optimal performance of design parameters obtained in terms of statistical and economic criteria, but in economic statistical design, statistical and economic criteria considered jointly. In this paper economic statistical design is derived based on minimizing average cost per time and by considering maximum values for the adjusted average time to signal ($AATS$) and average numbers of false alarm in the quality cycle (ANF). If we note the average cost of the cycle per time by $E(A)$ and the set of economic design parameters of $\overline{X}$ control charts by F, we can show the economic statistical design of $\overline{X}$ control charts as follows:

Minimize $E_F(A)$ subject to $AATS \leqslant AATS_U$ and $ANF \leqslant ANF_U$

where $AATS_u$ and ANF_u are the corresponding bounds of values of $AATS$ and ANF.

5 Real Example and Solution Procedure

Here, an example is presented to illustrate the solution procedure of the economic statistical design of the charts for correlated data (Chou and et al., 2001). A plant, located in central Taiwan, produces grape juice, which is contained in glass bottles. The target quantity of grape juice' is 200 cm^3 for each bottle. In the production process, the grape juice is inserted into twelve bottles at a time, and the twelve bottles of juice will be packed in a box later. Before the twelve bottles of grape juice are packed, the inspector samples the first four bottles to check whether the quantity of grape juice for each bottle is 200 cm^3 and $\overline{X}$ chart is applied to monitor the process of insertion. The subgroups, that is, the first four bottles from the recent 100 successive boxes, are viewed as a random sample from a multivariate distribution. Moreover, the vector of mean values is, the average correlation factor is estimated to be 0.1.

Some of the fixed parameters including cost parameters (Y, a, b) and time parameters (Z_0, Z_1) and Taguchi loss function parameters (A, σ, P, Δ) have been determined based on past experience. We also have ρ here as a fixed parameter. In the model w_i is non-fixed cost parameters and Z_{2i} is non-fixed time parameter. Weibull distribution parameters (λ_i, k), shift parameter δ_i and design parameters (n, h_1, L) are also presented in the model. In the numerical example, we assume: $Y = 1500,\ a = 25,\ b = 25,\ Z_0 = 1.35,\ Z_1 = 1.35,\ \rho = 0.1,\ A = 5,\ \sigma = 2,\ P = 50,\ \Delta = 0.5$. The above parameters are not affected by the occurrence of different assignable causes and the shift created in the mean process. In this paper, we need some assumptions to estimate other parameters.

a) D_0, D_{1i} is approximately computed based on modified Taguchi loss function.

b) Suppose that $\delta_i = 1.75$ is a base case. In this case, assume that $Z_{2i} = 2.5, w_i = 750$. Banerjee and Rahim (1988) single assignable cause model is compared with our multiple assignable causes model. Base case parameters are also considered for single assignable cause model $(w = 750, Z_2 = 2.5, \lambda = 0.02, \delta = 1.75, \rho = 0.1)$.

c) We assume that process is disturbed by ten assignable causes which produce ten shifts amount in the process mean vector that is from 0.75 to 3 with increments 0.25.

In this article, we noted the prior distribution for δ_i by PD_i. As mentioned earlier three distribution uniform, negative-exponential and half-normal are considered as a prior for δ_i. Based case are considered for $\delta_i = 1.75$. PD_5 is the notation for base case prior distribution. The amount of Weibull scale parameter are calculated by the use of prior distributions. Other parameter formulas are

$$W_i = (\frac{PD_i}{PD_5}) \times 750 \tag{34}$$

$$Z_{2i} = (\frac{PD_i}{PD_5}) \times 2.5 \tag{35}$$

$$\lambda_i = (\frac{PD_i}{PD_5}) \times \lambda_5. \tag{36}$$

Input parameters values are listed in Table 2. Determination of the optimal model parameters is performed by minimizing loss cost under constraint through the R software package Optim. By using this package general-purpose optimization based on Nelder and Mead (1965), quasi-Newton and conjugate-gradient algorithms is done.

Table 2. Model input parameters

		PD_i			Z_{2i}			w_i			λ_i		
A_i	δ_i	NE	Un	HN	NE	Un	HN	NE	Un	HN	NE	Un	HN
1	0.75	0.344	0.100	0.372	4.122	2.5	3.417	1237	750	1025	0.003	0.001	0.002
2	1	0.303	0.100	0.352	3.637	2.5	3.235	1091	750	971	0.002	0.001	0.002
3	1.25	0.268	0.100	0.328	3.210	2.5	3.016	963	750	905	0.002	0.001	0.002
4	1.5	0.236	0.100	0.301	2.833	2.5	2.767	850	750	830	0.002	0.001	0.002
5	1.75	0.208	0.100	0.272	2.500	2.5	2.500	750	750	750	0.002	0.001	0.002
6	2	0.184	0.100	0.242	2.206	2.5	2.224	662	750	667	0.001	0.001	0.001
7	2.25	0.162	0.100	0.212	1.947	2.5	1.947	584	750	584	0.001	0.001	0.001
8	2.5	0.143	0.100	0.183	1.718	2.5	1.678	515	750	504	0.001	0.001	0.001
9	2.75	0.126	0.100	0.155	1.516	2.5	1.424	455	750	427	0.001	0.001	0.001
10	3	0.112	0.100	0.130	1.338	2.5	1.190	401	750	357	0.001	0.001	0.001

Comparison between optimal values (n, h_1, L) and loss cost based on the economic statistical design for our multiplicity-cause model and single-cause model of Banerjee and Rahim (1988) are presented in Table 3 for different values of the shape parameter of the Weibull distribution. Table 3 shows that to achieve the desired statistical properties we need to pay a price for it. in many situations, may be a relatively small price to pay in order to achieve the improved statistical performance of the control. It should be noted that according to the values obtained in economic design, the upper limit of $AATS$ was considered 1 and the upper limit of ANF were considered 0.5.

Table 3. Optimal parameters obtained under multiple assignable causes by considering economic-statistical design

					Economic Statistical design				
k	PD	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$
	BR	6	1.44	3.003	0.014	0.84	1	0.47	169.28
1	NE	10	1.26	2.45	0.009	0.84	1	0.48	245.09
	Un	9	1.36	2.6	0.006	0.88	1	0.5	215.93
	HN	9	1.27	2.49	0.009	0.84	1	0.5	228.43
	BR	3	3.13	2.23	0.041	0.77	1	0.34	210.63
1.5	NE	7	5.26	1.96	0.039	0.85	1	0.18	165.43
	Un	4	5.3	2.08	0.032	0.81	1	0.25	149.4
	HN	6	5.51	1.91	0.045	0.85	1	0.21	158.82
	BR	5	3.88	2.62	0.026	0.85	1	0.1	219
1.8	NE	6	5.6	2.02	0.035	0.8	1	0.08	170.72
	Un	5	6.09	2.27	0.019	0.81	0.88	0.066	161.15
	HN	5	5.8	2.13	0.028	0.8	0.98	0.065	166.9
	BR	5	3.7	2.61	0.026	0.84	0.89	0.081	227.56
2	NE	5	5.45	2.02	0.036	0.79	0.89	0.059	177.3
	Un	5	5.66	2.23	0.019	0.81	0.72	0.052	169.42
	HN	5	5.4	2.08	0.031	0.8	0.8	0.057	173.76
	BR	4	3.2	2.6	0.027	0.83	0.69	0.06	237.35
2.5	NE	5	4.58	1.89	0.049	0.8	0.6	0.047	187.75
	Un	4	4.71	2.12	0.028	0.82	0.46	0.046	182.23
	HN	5	4.53	1.96	0.042	0.81	0.53	0.045	184.07
	BR	4	2.85	2.52	0.029	0.83	0.56	0.048	239.2
3	NE	5	3.93	1.74	0.07	0.81	0.44	0.042	192.73
	Un	4	4.01	2.03	0.036	0.82	0.33	0.04	188.57
	HN	5	3.89	1.83	0.058	0.19	0.38	0.045	188.93
	BR	4	2.38	2.43	0.033	0.83	0.42	0.037	235.92
4	NE	4	3.13	1.42	0.13	0.83	0.3	0.036	195.93
	Un	4	3.16	1.85	0.05	0.83	0.22	0.032	193.18
	HN	4	3.09	1.57	0.1	0.83	0.27	0.034	192.04

In Table 4, the comparison between optimal values and loss cost for economic statistical design by considering the non-uniform sampling and uniform sampling scheme for different values of the Weibull distribution shape parameter is given. As it is seen in Table 4 when we use economic statistical design, the loss cost becomes greater when uniform sampling scheme is used instead of non-uniform sampling scheme.

Table 4. Optimal parameters obtained under ESD by considering non-uniform and uniform schem

		Economic Statistical design with non-uniform sampling scheme								Economic Statistical design with uniform sampling scheme							
k	PD	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$	n	h_1	L	α	1-β	$AATS$	ANF	$E(A)$
	BR	6	1.44	3.003	0.014	0.84	1	0.47	169.28	6	1.45	3.01	0.014	0.85	1	0.46	169.51
1	NE	10	1.26	2.45	0.009	0.84	1	0.48	245.09	25	1.7	2.51	0.006	0.94	0.99	0.23	395.67
	Un	9	1.36	2.6	0.006	0.88	1	0.5	215.93	20	1.73	2.64	0.005	0.95	0.98	0.27	330.01
	HN	9	1.27	2.49	0.009	0.84	1	0.5	228.43	7	0.99	2.6	0.007	0.88	1	0.5	232.1
	BR	3	3.13	2.23	0.041	0.77	1	0.34	210.63	5	1.57	2.6	0.029	0.89	1	0.21	208.69
1.5	NE	7	5.26	1.96	0.039	0.85	1	0.18	165.43	7	1.3	1.89	0.047	0.84	1	0.5	238.72
	Un	4	5.3	2.08	0.032	0.81	1	0.25	149.4	6	1.31	2.19	0.023	0.86	0.99	0.33	217.77
	HN	6	5.51	1.91	0.045	0.85	1	0.21	158.82		27	1.56	3.5	0.0002	0.91	1	461.27
	BR	5	3.88	2.62	0.026	0.85	1	0.1	219	5	1.68	2.59	0.028	0.87	1	0.11	231.11
1.8	NE	6	5.6	2.02	0.035	0.8	1	0.08	170.72	26	1.79	2.23	0.014	0.95	1	0.06	492.58
	Un	5	6.09	2.27	0.019	0.81	0.88	0.066	161.15	29	1.84	2.36	0.009	0.97	0.98	0.054	589.85
	HN	5	5.8	2.13	0.028	0.8	0.98	0.065	166.9	27	1.81	2.24	0.013	0.97	0.97	0.06	527.9
	BR	5	3.7	2.61	0.026	0.84	0.89	0.081	227.56	5	1.5	2.56	0.029	0.87	0.98	0.1	244.21
2	NE	5	5.45	2.02	0.036	0.79	0.89	0.059	177.3	7	1.27	2	0.036	0.84	1	0.18	260.88
	Un	5	5.66	2.23	0.019	0.81	0.72	0.052	169.42	6	1.31	2.15	0.026	0.86	1	0.16	249.46
	HN	5	5.4	2.08	0.031	0.8	0.8	0.057	173.76	6	1.31	1.95	0.041	0.85	1	0.2	254.16
	BR	4	3.2	2.6	0.027	0.83	0.69	0.06	237.35	5	1.24	2.51	0.032	0.86	0.8	0.09	264.57
2.5	NE	5	4.58	1.89	0.049	0.8	0.6	0.047	187.75	5	1.05	2.02	0.036	0.88	1	0.14	269.17
	Un	4	4.71	2.12	0.028	0.82	0.46	0.046	182.23	5	1.1	2.19	0.024	0.82	1	0.11	265.68
	HN	5	4.53	1.96	0.042	0.81	0.53	0.045	184.07	36	1.58	3.95	0.0003	0.92	0.98	0.005	581.4
	BR	4	2.85	2.52	0.029	0.83	0.56	0.048	239.2	5	1.09	2.484	0.033	0.86	0.71	0.084	275.29
3	NE	5	3.93	1.74	0.07	0.81	0.44	0.042	192.73	5	1.02	2.02	0.036	0.87	1	0.1	277.25
	Un									5	1.2	2.1	0.03	0.84	0.98	0.089	282.59
	HN	4	4.01	2.03	0.036	0.82	0.33	0.04	188.57	6	1.29	1.88	0.049	0.84	1	0.11	280.85
	BR	5	3.89	1.83	0.058	0.19	0.38	0.045	188.93	4	0.92	2.44	0.035	0.86	0.61	0.07	304.26
4	NE	4	2.38	2.43	0.033	0.83	0.42	0.037	235.92	5	1.01	2.04	0.035	0.87	1	0.07	286.27
	Un	4	3.13	1.42	0.13	0.83	0.3	0.036	195.93	4	4.08	2.2	0.024	0.79	0.84	0.07	293.88
	HN	4	3.16	1.85	0.05	0.83	0.22	0.032	193.18	5	0.96	2.11	0.03	0.77	0.96	0.066	287.06

6 The Effect of Changing Model Parameters

We explore the effect of input parameters on the calculations of design parameters and loss cost by using response surface methodology (RSM). Response surface method (RSM) designs help us quantify the relationships between one or more measured responses and the vital input parameters. In this method, the relations between parameters, considered totally. Fixed input parameters are given in Table 5.

Table 5. Fixed model parameters range

Name	Units	Type	Low Actual	High Actual	Low Coded	High Coded	Mean	Std. Dev.
K		Numeric	1	4	-1	1	2.489	1.406
Z_0	Hour	Numeric	0.25	1.35	-1	1	0.79	0.515
Z_1	Hour	Numeric	0.25	1.35	-1	1	0.822	0.516
a	Dollar	Numeric	20	100	-1	1	57.762	37.704
b	Dollar	Numeric	1	30	-1	1	15.705	13.754
A	Dollar	Numeric	1	5	-1	1	3.134	1.872
y	Dollar	Numeric	300	1500	-1	1	901.01	559.127
r		Numeric	0	1	-1	1	0.483	0.473
$sigma$		Numeric	1	5	-1	1	2.931	1.893
p		Numeric	10	100	-1	1	53.946	42.567
$delta$		Numeric	0.1	1	-1	1	0.539	0.423

In Table 6, the tests (runs) for various combinations of fixed model input parameters of a planned experiment using D-optimal outline is given. The optimal values for design parameters and loss cost connected with them are presented in Table 6. In this section, only negative exponential distribution is used as a prior for δ_i. The related package give us Analysis of variance (ANOVA) results to analyze the chosen model. We compare p-values in the anova results with significant level and then judge about the relations between input and design parameters. In Table 7, p-values were reported and one can judge about the relations between parameters by that. Design Expert (we use this package for response surface method) provides prediction equations in terms of actual model input parameters and design parameters and loss cost (formulas (34-37)). We also present the way of relations between input and design parameters (decreasing or increasing) according theses formulas.

Table 6. The optimal parameters got by taking different combinations of fixed time and cost parameters

Run	K	Z_0	Z_1	a	b	D_0	Y	ρ	σ	P	Δ	n	h_1	L	$E(A)$
1	1.00	0.25	1.35	20.00	30.00	5.00	1500.00	0.00	1.00	10.00	0.10	11	1.30215	2.55756	270.695
2	1.00	1.35	0.25	74.13	30.00	1.00	1462.50	0.02	5.00	10.00	1.00	29	1.89275	2.37332	536.061
3	1.00	1.07	1.35	20.00	17.74	1.00	1500.00	0.00	1.00	10.00	1.00	30	1.94339	2.5964	281.897
4	1.00	1.35	1.35	100.00	30.00	1.00	1500.00	1.00	1.00	10.00	0.10	48	1.67787	1.68414	880.07
5	1.00	1.02	0.25	20.00	30.00	5.00	1500.00	0.00	1.00	100.00	0.43	11	1.30226	2.55829	280.064
6	4.00	0.25	0.25	59.34	30.00	5.00	1500.00	1.00	5.00	10.00	1.00	6	2.49973	1.75023	386.81
7	1.38	0.25	1.35	20.00	1.00	5.00	300.00	1.00	1.00	10.00	1.00	21	4.63033	2.88168	59.3397
8	1.00	0.25	0.86	20.00	1.00	5.00	300.00	0.00	1.00	100.00	0.10	29	1.93292	2.67074	38.6351
9	1.00	1.35	0.25	20.00	14.73	5.00	815.61	1.00	1.00	10.00	0.10	50	1.62729	2.37072	456.866
10	4.00	1.35	1.35	100.00	1.00	1.00	300.00	0.00	1.00	100.00	1.00	9	3.27327	1.82437	157.23
11	1.00	1.35	1.35	100.00	1.00	5.00	1500.00	0.00	1.00	100.00	1.00	28	1.67509	3.54884	128.689
12	4.00	0.52	1.35	25.70	15.28	2.81	300.00	0.00	5.00	100.00	0.10	2	3.47358	1	148.714
13	1.00	1.35	1.35	20.00	30.00	3.04	300.00	0.32	1.00	68.35	0.64	8	1.09431	2.43863	241.582
14	1.00	0.80	1.35	100.00	1.00	5.00	1230.74	1.00	5.00	61.65	0.10	50	1.41605	3.77037	128.391
15	1.00	0.97	0.25	100.00	30.00	5.00	514.58	0.00	1.00	10.00	0.10	13	1.52744	2.50126	322.657
16	2.69	1.35	0.25	20.00	30.00	2.51	300.00	0.00	5.00	100.00	1.00	7	2.35738	1.71054	715.689
17	4.00	0.25	0.25	20.00	30.00	1.00	1500.00	0.00	5.00	10.00	0.53	2	3.94811	1	183.894
18	1.00	0.25	0.47	20.00	30.00	2.40	1318.10	1.00	1.00	10.00	1.00	7	1.12897	2.24495	231.928
19	1.00	1.35	1.35	20.00	1.00	5.00	300.00	0.00	5.00	10.00	0.10	29	1.90034	2.83148	38.913
20	1.00	0.25	1.35	20.00	1.00	1.00	300.00	0.00	5.00	74.53	1.00	21	1	3.42575	151.747
21	1.00	0.72	0.55	100.00	1.00	1.00	1500.00	0.00	5.00	100.00	0.60	28	1.47442	3.92026	162.946
22	1.00	1.30	1.35	100.00	30.00	1.00	300.00	1.00	5.00	100.00	1.00	12	1.16974	2.71639	576.474
23	1.00	0.25	0.25	20.00	30.00	5.00	300.00	0.00	5.00	10.00	1.00	11	1.29898	2.55879	368.039
24	4.00	0.25	1.35	100.00	1.00	1.00	1500.00	0.00	5.00	10.00	0.10	19	4.9403	1.81954	127.992
25	2.82	0.25	0.65	20.00	13.79	1.00	300.00	1.00	1.00	100.00	1.00	3	4.4068	1.26713	153.072
26	4.00	1.35	1.35	100.00	30.00	5.00	1500.00	0.00	3.82	23.27	1.00	6	2.50719	1.73093	371.108
27	1.00	0.25	0.25	30.94	1.00	1.00	300.00	1.00	5.00	10.00	1.00	37	1.12904	4.98654	85.5081
28	4.00	1.35	0.25	20.00	30.00	5.00	300.00	0.00	1.00	10.00	1.00	2	3.84029	1	173.942
29	4.00	1.35	0.25	20.00	1.00	1.00	1500.00	1.00	1.00	10.00	1.00	15	4.60579	2.68295	140.685
30	4.00	0.25	1.35	20.00	1.00	5.00	1500.00	1.00	1.00	100.00	0.10	19	4.7345	2.26292	119.119
31	1.00	1.35	1.35	20.00	1.00	1.00	300.00	1.00	5.00	100.00	0.34	49	1.23458	5	90.7123
32	1.00	0.25	0.25	100.00	19.47	1.00	1500.00	1.00	3.42	10.00	0.10	6	1.02531	2.32079	226.603
33	4.00	1.35	1.35	100.00	29.09	4.38	697.39	0.00	1.00	97.19	0.10	8	4.93353	1.19844	157.599
34	3.97	1.35	0.49	100.00	1.00	5.00	1350.00	1.00	1.00	100.00	0.10	22	4.83426	1.82485	148.779
35	2.09	0.25	1.35	100.00	30.00	2.79	1500.00	0.54	5.00	10.00	1.00	6	4.46577	1.91817	299.636
36	4.00	1.35	1.35	69.70	1.00	5.00	300.00	1.00	1.00	10.00	0.10	24	4.83772	1.49777	124.386
37	4.00	1.06	0.25	20.00	1.00	1.00	300.00	0.00	3.48	10.00	0.10	21	4.91675	2.3929	136.142
38	3.43	0.25	0.25	100.00	1.00	1.54	300.00	1.00	5.00	100.00	0.10	14	5.04971	2.86406	158.044
39	1.00	1.35	0.25	100.00	30.00	5.00	1500.00	1.00	5.00	100.00	1.00	12	1	2.20222	1296.3
40	4.00	1.35	0.25	100.00	1.00	5.00	693.08	0.00	5.00	10.00	1.00	17	2.25976	2.96135	286.358
41	4.00	1.35	0.25	20.00	5.66	1.00	1500.00	1.00	5.00	100.00	0.10	8	4.55457	2.28683	153.244
42	1.00	1.35	0.82	100.00	1.00	3.86	300.00	1.00	3.28	10.00	1.00	49	1.23564	5	164.118
43	4.00	1.35	0.25	100.00	30.00	4.87	300.00	1.00	5.00	10.00	0.10	7	4.77231	1.25906	187.977
44	4.00	0.34	1.12	20.00	1.00	4.23	1500.00	0.00	5.00	10.00	1.00	17	1.99966	3.25761	196.216

Table 6. (continued)

Run	K	Z_0	Z_1	a	b	D_0	Y	ρ	σ	P	Δ	n	h_1	L	$E(A)$
45	1.00	1.35	0.25	74.53	1.00	1.00	1500.00	1.00	1.00	100.00	1.00	50	1.31109	4.48334	115.586
46	2.51	1.35	0.82	85.41	17.07	1.00	300.00	0.33	5.00	50.89	0.10	10	6.75992	1	133.78
47	4.00	1.35	1.35	100.00	1.00	5.00	1500.00	1.00	5.00	100.00	1.00	33	1.67409	5	570.623
48	1.00	0.46	1.35	20.00	23.47	1.00	1500.00	1.00	4.94	100.00	0.97	49	1.24214	4.79883	1115.2
49	4.00	0.25	1.35	100.00	30.00	5.00	724.89	1.00	1.00	0.10	0.10	2	4.6476	1	137.464
50	4.00	0.25	1.35	100.00	1.00	5.00	300.00	0.00	2.98	64.46	0.64	17	2.27403	2.89317	236.29
51	4.00	1.35	1.35	20.00	1.00	5.00	1500.00	0.00	1.00	50.75	0.10	21	4.9021	2.50352	117.57
52	1.00	1.35	0.25	100.00	1.00	5.00	300.00	0.00	4.09	100.00	0.10	29	1.88527	2.94159	88.9871
53	4.00	0.25	0.25	100.00	30.00	5.00	1500.00	0.00	5.00	100.00	0.10	3	3.73431	1	219.918
54	4.00	0.25	0.25	43.85	1.00	1.00	1500.00	0.00	2.49	100.00	1.00	17	2.30593	3.16183	219.259
55	1.00	0.25	0.25	100.00	8.08	2.99	300.00	0.00	1.00	100.00	1.00	16	1.69484	2.46356	169.738
56	1	1.02	0.25	20.00	30.00	5.00	1500.00	0.00	1.00	100.00	0.43	11	1.30226	2.55829	280.064
57	4	0.69	0.25	50.06	30.00	1.00	1500.00	0.65	1.00	10.00	0.10	2	4.65149	1	150.111
58	1	1.35	1.35	20.00	30.00	1.00	1500.00	0.00	5.00	100.00	0.10	12	1.47347	2.51426	273.706
59	1	0.25	1.35	100.00	1.00	1.00	300.00	0.62	1.00	10.00	0.10	50	1.64956	2.43712	98.3792
60	4	0.25	0.93	20.00	1.00	5.00	694.99	0.90	5.00	10.00	0.10	12	4.55665	2.92537	127.551
61	1	1.35	1.35	100.00	1.00	5.00	1500.00	0.00	1.00	100.00	1.00	28	1.67509	3.54884	128.689
62	4	1.35	0.25	100.00	9.65	2.58	1500.00	0.00	1.00	10.00	0.10	14	4.89502	1.49059	165.582
63	1	1.35	0.25	47.18	1.00	5.00	1500.00	0.00	5.00	10.00	0.10	29	1.8613	3.04018	55.0029
64	4	0.25	1.35	100.00	30.00	1.00	1500.00	1.00	5.00	100.00	0.29	3	3.36542	1	208.412
65	1	1.35	1.35	20.00	30.00	5.00	1500.00	1.00	5.00	10.00	1.00	7	1.12835	2.24429	340.112
66	4	0.25	1.35	20.00	30.00	5.00	866.07	0.00	1.00	100.00	1.00	4	2.82212	1.4069	230.987
67	1.51	0.56	1.35	20.00	30.00	1.00	300.00	1.00	5.00	10.00	0.10	6	5.94585	1	100.274
68	4	0.25	0.73	100.00	30.00	1.00	300.00	0.00	1.00	10.00	1.00	7	4.92339	1.20122	172.767
69	4	1.35	0.25	20.00	1.00	5.00	377.94	1.00	1.00	100.00	1.00	12	2.15721	2.43705	198.868
70	3.3	1.35	1.35	20.00	30.00	5.00	300.00	1.00	3.62	100.00	0.10	2	4.13535	1	144.953
71	4	1.35	0.25	20.00	30.00	1.00	300.00	0.00	1.00	100.00	0.10	8	4.90599	1.2635	171.111
72	4	1.35	1.35	20.00	30.00	1.00	300.00	0.59	5.00	10.00	1.00	2	2.67896	1	185.09
73	1	0.25	0.25	20.00	1.00	5.00	1500.00	1.00	5.00	100.00	1.00	34	1	5	734.137
74	4	0.25	0.93	20.00	1.00	5.00	694.99	0.90	5.00	10.00	0.10	12	4.55665	2.92537	127.551
75	1.97	0.25	1.35	100.00	30.00	1.00	1500.00	0.00	1.00	100.00	0.10	7	6.61145	1.78959	132.479
76	2.48	0.25	0.25	100.00	1.00	5.00	1500.00	1.00	1.00	10.00	0.53	6	5	3	132.463
77	1	0.25	0.99	72.13	30.00	5.00	300.00	1.00	5.00	100.00	0.65	9	1	2.25231	745.989
78	4	0.25	0.25	20.00	30.00	5.00	300.00	1.00	1.00	59.35	0.10	7	4.78219	1.22387	170.418
79	1	0.25	0.25	20.00	30.00	1.00	300.00	0.51	5.00	100.00	0.10	8	1.12311	2.34099	238.287
80	1	0.25	0.25	20.00	1.00	1.00	1500.00	0.00	1.00	47.10	0.10	29	1.82902	3.16842	39.5404
81	4	0.25	1.35	100.00	1.00	1.00	1298.00	1.00	1.00	49.69	1.00	8	4.06379	1.91795	148.766
82	75	0.25	1.35	100.00	30.00	5.00	300.00	0.00	5.00	10.00	0.10	5	6.35491	1.47663	119.654
83	4	1.35	1.35	20.00	30.00	1.27	1500.00	1.00	1.00	100.00	1.00	3	3.47689	1	173.252
84	1	0.25	1.35	100.00	1.00	1.00	300.00	0.62	1.00	10.00	0.10	50	1.64956	2.43712	98.3792
85	2.04	0.25	1.35	50.05	1.00	2.51	883.88	0.00	2.83	10.00	0.52	14	6.32552	2.97878	97.3265
86	4	0.91	0.25	100.00	30.00	5.00	300.00	0.52	1.95	100.00	1.00	4	2.25438	1	431.264
87	2.15	1.35	0.25	100.00	30.00	1.00	300.00	1.00	1.00	10.00	1.00	4	5.6451	2.00752	146.698
88	4	0.25	1.35	20.00	30.00	5.00	866.07	0.00	1.00	100.00	1.00	4	2.82212	1.4060	230.987

Table 7. Checking fixed time and cost parameters significant relation with design parameters

Source	p-value(n)	p-value(h_1)	p-value(L)	p-value(E(A))
K	<0.0001	<0.0001	<0.0001	0.0212
Z_0	0.0152	0.4795	0.8604	0.3533
Z_1	0.2316	0.4244	0.5194	0.4899
a	0.5388	0.2787	0.31	0.306
b	<0.0001	0.5477	<0.0001	<0.0001
A	0.4329	0.0499	0.1823	0.1726
Y	0.3955	0.226	0.0169	0.0284
ρ	0.0927	0.6862	0.128	0.0213
sigma	0.6299	0.1311	0.0019	0.0057
P	0.4701	0.0385	0.1632	0.0476
delta	0.4673	0.0002	0.0001	0.0023

The linear model between design parameters (n, h_1, L) and the parameters of cost and time are as follows.

$$\hat{n} = 32.28 - 5.05k - +4.83Z_0 - 0.56b \tag{37}$$

$$\hat{h_1} = 2.86 + 0.69k - 0.13A - 0.006P - 1.19\Delta \tag{38}$$

$$\hat{L} = 2.93 - 0.36k - 0.047b + 0.0026Y + 0.1\sigma + 0.59\Delta. \tag{39}$$

The linear model between loss cost and the parameters of cost and time is as follows.

$$\hat{E}(A) = -163.4 - 37.52k + 5.73b + 0.074Y + 92.4\rho + 28\sigma + 0.88P + 139.2\Delta. \tag{40}$$

6.1 The Effect of Changes in Taguchi Loss Function Parameters

As seen in Table 7 and formula (34-37), the increase in parameter A leads to decrease in optimal values of h_1 significantly. The increase in P leads to decrease in the optimal values of h_1 and increase in the optimal value of $E(A)$. The increase in Δ leads to decrease in h_1 leads to decrease in h_1 and increase in L and $E(A)$. The increase in Δ leads to increase in L and $E(A)$.

6.2 The Effect of Changes in ρ Parameter

Table 7 shows the effect of changing the correlation coefficient on the optimal economic statistical design parameters of $\overline{X}$ chart. As the measurements in the sample are positively correlated (which means the measurements are more homogeneous in the positive direction) we put correlation coefficient value between 0 and 1. As seen in Table 7, changes in the ρ parameter has no effect on n, h_1 and L. The increase in ρ leads to increase in optimal values of $E(A)$.

6.3 The Effect of Changes in Cost Parameters

As seen in Table 7 and formula (34-37), the increase in unit sample cost b leads to decrease in optimal values of n and L and increase the optimal values of $E(A)$ significantly. The increase in Y leads to incraese in the optimal values of L and $E(A)$.

In Table 8, standard values of are taken from Table 2. The numerical results show that if the amounts of w_i are doubled or quadrupled in general the optimum value of n, h_1 and L will not change significantly. It is also observed that optimum values of $E(A)$ increase. The results of this study illustrated in Figure 3.

Figure 3. The relationship between parameter w_i and design parameters and loss cost.

Table 8. Changes in optimal design parameters with variations in w_i

	Standard w_i				Two times w_i				Four times w_i			
Run	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$
1	11	1.30215	2.55756	270.695	11	1.279878	2.564017	282.2976	28	1.948412	2.422471	470.1238
2	29	1.89275	2.37332	536.061	29	1.892748	2.373309	548.7442	29	1.892743	2.37329	574.1108
3	30	1.94339	2.5964	281.897	30	1.943229	2.596996	294.4282	30	1.943395	2.59639	319.4908
4	48	1.67787	1.68414	880.07	48	1.678087	1.680843	892.307	48	1.678529	1.674176	916.772
5	11	1.30226	2.55829	280.064	12	1.419033	2.527865	294.7495	12	1.420948	2.527263	320.2057
6	6	2.49973	1.75023	386.81	6	2.513223	1.748876	533.695	6	2.542211	1.749018	827.2713
7	21	4.63033	2.88168	59.3397	12	4.575557	2.189769	92.65219	23	4.624591	2.971574	168.8763
8	29	1.93292	2.67074	38.6351	29	1.898317	2.852354	50.89864	29	1.903723	2.826383	76.27101
9	50	1.62729	2.37072	456.866	50	1.627292	2.370722	469.64	7	1.099225	2.265749	162.7131
10	9	3.27327	1.82437	157.23	11	3.744236	2.588129	277.2244	12	4.335868	3.418338	499.6959
11	28	1.67509	3.54884	128.689	24	1.65677	3.291533	138.4841	28	1.652305	3.595943	166.387
12	2	3.47358	1	148.714	5	4.648633	1.796927	268.9556	12	4.801749	2.277311	503.5333
13	8	1.09431	2.43863	241.582	7	1	2.468689	241.3243	7	1	2.468821	266.2766
14	50	1.41605	3.77037	128.391	50	1.423625	3.718411	141.2124	50	1.36983	4.075713	165.5144
15	13	1.52744	2.50126	322.657	12	1.40853	2.536657	334.4439	13	1.524734	2.502031	360.6387
16	7	2.35738	1.71054	715.689	7	2.169654	1.821615	831.3295	8	2.243149	1.777794	1048.084
17	2	3.94811	1	183.894	5	4.742974	1.602074	326.8843	8	4.817565	1.781426	598.9789
18	7	1.12897	2.24495	231.928	6	1	2.339676	6	1	220.4674	2.339642	245.9286
19	29	1.90034	2.83148	38.913	29	1.873418	2.961741	51.34353	29	1.928025	2.700847	76.82097
20	21	1	3.42575	151.747	21	1	3.429624	164.3967	21	1	3.437693	189.6963
21	28	1.47442	3.92026	162.946	29	1.489242	3.911925	175.7532	28	1.49375	3.886804	201.0156
22	12	1.16974	2.71639	576.474	8	1.195967	2.1948	525.2868	49	1.291194	4.546609	1482.818
23	11	1.29898	2.55879	368.039	11	1.321071	2.553116	380.7585	11	1.382221	2.536622	407.5116
24	19	4.9403	1.81954	127.992	19	4.981451	1.921633	237.8625	20	4.981489	1.935669	458.1367
25	3	4.4068	1.26713	153.072	4	5.132223	1.59604	259.69	7	5.596968	2.157698	467.0672
26	6	2.50719	1.73093	371.108	6	2.506415	1.64337	492.6398	5	2.504687	1.245952	733.3938
27	37	1.12904	4.98654	85.5081	34	1.095722	5	96.83884	34	1.097386	5	122.5781
28	2	3.84029	1	173.942	6	4.792063	1.576854	320.4878	10	4.898981	1.695621	593.0345
29	15	4.60579	2.68295	140.685	18	4.709604	2.610177	269.4833	27	3.107033	2.053517	593.3217
30	19	4.7345	2.26292	119.119	23	4.754049	2.050573	231.4279	27	3.107051	2.053525	498.8167
31	49	1.23458	5	90.7123	49	1.23452	5	103.3031	49	1.234764	5	128.4761
32	6	1.02531	2.32079	226.603	8	1.18153	2.205554	247.2713	7	1.109283	2.259062	270.0853
33	8	4.93353	1.19844	157.599	10	4.948013	1.365032	276.3673	12	4.940348	1.483479	505.2788
34	22	4.83426	1.82485	148.779	23	25.01167	5	4.72E+08	24	4.802763	1.995399	521.9696
35	6	4.46577	1.91817	299.636	6	4.512503	1.927853	375.872	6	4.615541	1.952773	528.077
36	24	4.83772	1.49777	124.386	25	4.813092	1.619114	235.3143	27	4.738764	2.073906	458.2854
37	21	4.91675	2.3929	136.142	19	4.98071	1.915167	263.8216	21	3.64997	2.324985	573.9605
38	14	5.04971	2.86406	158.044	17	5.091882	2.825689	279.7487	20	5.187168	2.564239	524.3298
39	12	1	2.20222	1296.3	12	1	2.201754	1308.998	12	1	2.198546	1334.582
40	17	2.25976	2.96135	286.358	17	2.270278	2.915652	434.8279	8	2.218375	1.183406	711.3921
41	8	4.55457	2.28683	153.244	12	4.463948	2.469552	289.5325	16	4.67428	2.420258	549.4388
42	49	1.23564	5	164.118	49	1.238105	4.982363	176.9167	49	1.238292	4.980142	202.2811
43	7	4.77231	1.25906	187.977	10	4.780118	1.390928	333.1482	12	4.771244	1.440395	601.774
44	17	1.99966	3.25761	196.216	17	2.012113	3.277059	325.7667	17	2.041333	3.340505	584.7111

Table 8. (continued)

	Standard w_i				Two times w_i				Four times w_i			
Run	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$
45	50	1.31109	4.48334	115.586	50	1.327409	4.368551	128.3377	50	1.337436	4.302803	153.9546
46	10	6.75992	1	133.78	2	6.17512	1	210.5141	13	6.794761	1.049818	416.4213
47	33	1.67409	5	570.623	33	1.675088	5	696.3047	33	1.677104	5	947.6447
48	49	1.24214	4.79883	1115.2	49	1.242144	4.798809	1127.786	49	1.242146	4.798774	1152.963
49	2	4.6476	1	137.464	11	4.819419	1.21287	283.9704	13	4.837118	1.335953	514.3267
50	17	2.27403	2.89317	236.29	17	2.286329	2.926344	360.9767	17	2.314923	3.031806	610.1963
51	21	4.9021	2.50352	117.57	19	4.977535	1.969476	227.2635	20	4.913828	1.976199	449.7949
52	29	1.88527	2.94159	88.9871	20	1.838977	2.4533	97.80908	21	1.856554	2.431206	123.0765
53	3	3.73431	1	219.918	3	3.92587	1	358.2317	3	4.74501	1.001444	621.999
54	17	2.30593	3.16183	219.259	17	2.337837	3.21187	367.4818	17	2.434576	3.478481	663.0504
55	16	1.69484	2.46356	169.738	16	1.678821	2.545796	185.0354	17	1.744992	2.453554	210.1927
56	11	1.30226	2.55829	280.064	12	1.419033	2.527865	294.7495	12	1.420948	2.527263	320.2057
57	2	4.65149	1	150.111	12	4.859885	1.164075	323.8069	5	4.806158	1	547.4716
58	12	1.47347	2.51426	273.706	10	1.264097	2.56833	282.0416	12	1.454488	2.518831	310.6873
59	50	1.64956	2.43712	98.3792	50	1.649559	2.437125	110.965	50	1.64956	2.437125	136.1365
60	12	4.55665	2.92537	127.551	18	4.706903	2.605169	245.4956	27	3.129706	2.064853	531.972
61	28	1.67509	3.54884	128.689	24	1.65677	3.291533	138.4841	28	1.652305	3.595943	166.387
62	14	4.89502	1.49059	165.582	15	4.957221	1.614781	295.7836	18	4.980312	1.800286	555.4114
63	29	1.8613	3.04018	55.0029	29	1.891034	2.914944	68.15962	29	1.865946	3.024224	93.36678
64	3	3.36542	1	208.412	3	3.442154	1	328.2013	3	3.670664	1	566.1021
65	7	1.12835	2.24429	340.112	10	1.251015	2.153268	370.9563	7	1.131655	2.24238	377.8463
66	4	2.82212	1.4069	230.987	4	2.851375	1.37969	353.2406	4	2.917857	1.318701	597.4257
67	6	5.94585	1	100.274	6	5.945763	1	144.1603	6	5.94575	1	231.9318
68	7	4.92339	1.20122	172.767	10	4.803682	1.3959	307.8552	12	4.925987	1.827568	553.2778
69	12	2.15721	2.43705	198.868	7	1.950052	1	344.9134	6	1.950069	1	612.752
70	2	4.13535	1	144.953	3	5.164277	1.304838	252.1456	5	5.335766	1.39984	463.1076
71	8	4.90599	1.2635	171.111	10	4.893497	1.332623	308.2784	14	4.929884	1.673936	586.4289
72	2	2.67896	1	185.09	2	2.518295	1	303.0705	2	2.247487	1	531.9742
73	34	1	5	734.137	34	1	5	746.9561	34	1	5	772.594
74	12	4.55665	2.92537	127.551	18	4.706903	2.605169	245.4956	27	3.129706	2.064853	531.972
75	7	6.61145	1.78959	132.479	7	6.621062	1.800816	201.2021	7	6.654329	1.811589	338.4821
76	6	5	3	132.463	6	5	3	229.4932	6	5	3	423.5546
77	9	1	2.25231	745.989	9	1	2.255112	758.588	9	1.140707	2.215873	783.5198
78	7	4.78219	1.22387	170.418	10	4.794045	1.343289	314.4857	12	4.796857	1.378195	580.8989
79	8	1.12311	2.34099	238.287	6	1	2.419048	233.398	49	1.733926	1.807348	908.5372
80	29	1.82902	3.16842	39.5404	29	1.815094	3.220183	52.37013	29	1.825165	3.176689	77.90826
81	8	4.06379	1.91795	148.766	15	4.547075	2.816671	260.6196	17	4.671691	2.547892	483.1414
82	5	6.35491	1.47663	119.654	5	6.373371	1.493874	177.8011	5	6.38058	1.499075	294.4994
83	3	3.47689	1	173.252	2	3.599429	1	291.6996	5	4.388229	2.222954	526.413
84	50	1.64956	2.43712	98.3792	50	1.649559	2.437125	110.965	50	1.64956	2.437125	136.1365
85	14	6.32552	2.97878	97.3265	14	6.288273	2.98573	168.8991	14	6.504645	2.839134	312.0071
86	4	2.25438	1	431.264	4	2.2014	1	570.8527	3	2.170457	1	847.3299
87	4	5.6451	2.00752	146.698	2	6.2128	1	206.002	2	6.195457	1	370.2503
88	4	2.82212	1.4060	230.987	4	2.851375	1.37969	353.2406	4	2.917857	1.318701	597.4257

6.4 The Effect of Changes in Time Parameters

As seen in Table 7, changes in the time parameter Z_1 has no effect on design parameters and $E(A)$. Increases in Z_0 leads to increases in optimal values of n.

In Table 9, standard values of Z_{2i} are taken from Table 2. The numerical results show that, if the amounts of Z_{2i} are doubled or quadrupled, in general, the optimum values of n, L and h_1 will not change significantly. The results of this study with regard to the negative exponential prior distribution are given in Table 9 and Figure 4.

Figure 4. The relationship between parameter Z_{2i} and design parameters and loss cost.

Table 9. Changes in optimal design parameters with variations in Z_{2i}

	Standard z_{2i}				Two times z_{2i}				Four times z_{2i}			
Run	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$
1	11	1.30215	2.55756	270.695	14	1.615967	2.481632	27.46484	13	1.481363	2.512339	243.7101
2	29	1.89275	2.37332	536.061	29	1.893654	2.373131	514.093	29	1.909601	2.369579	471.837
3	30	1.94339	2.5964	281.897	30	1.94338	2.596398	270.596	30	1.943374	2.596387	250.4956
4	48	1.67787	1.68414	880.07	48	1.681486	1.670671	844.5749	14	1.399301	2.036934	325.3492
5	11	1.30226	2.55829	280.064	11	1.302823	2.558131	268.6971	11	1.381419	2.537217	249.269
6	6	2.49973	1.75023	386.81	6	2.488514	1.752013	259.6434	6	2.479723	1.753859	156.5844
7	21	4.63033	2.88168	59.3397	16	4.407634	2.848623	51.25727	22	4.633361	2.90499	43.66854
8	29	1.93292	2.67074	38.6351	29	1.877202	2.94973	36.71491	28	1.879164	2.920125	33.82133
9	50	1.62729	2.37072	456.866	50	1.627274	2.370726	438.2113	50	1.627241	2.370733	405.1022
10	9	3.27327	1.82437	157.23	10	3.211719	1.890294	112.0263	11	3.163171	1.957034	71.06444
11	28	1.67509	3.54884	128.689	28	1.624544	3.616837	123.1558	21	1.648104	3.018126	110.8188
12	2	3.47358	1	148.714	2	3.392867	1	106.3865	3	3.336946	1	67.72656
13	8	1.09431	2.43863	241.582	7	1	2.468839	219.7505	7	1	2.468854	203.5038
14	50	1.41605	3.77037	128.391	50	1.463376	3.310011	128.1711	50	1.409532	3.796396	114.0646
15	13	1.52744	2.50126	322.657	14	1.557998	2.494057	311.0898	29	1.958204	2.411777	430.9765
16	7	2.35738	1.71054	715.689	7	2.238585	1.780647	530.5981	7	2.261592	1.766861	347.3685
17	2	3.94811	1	183.894	2	3.831104	1	126.2533	2	3.757521	1	77.43948
18	7	1.12897	2.24495	231.928	6	1	2.339711	199.2825	48	1.681067	1.686938	782.1585
19	29	1.90034	2.83148	38.913	29	1.873075	2.971886	37.27197	29	1.920526	2.753014	34.78894
20	21	1	3.42575	151.747	21	1	3.423897	145.6072	21	1	3.420665	134.6989
21	28	1.47442	3.92026	162.946	28	1.474052	3.864306	155.7072	28	1.488236	3.897491	144.4243
22	12	1.16974	2.71639	576.474	8	1.168555	2.215287	491.9783	8	1.190228	2.198708	455.4451
23	11	1.29898	2.55879	368.039	12	1.399137	2.533209	354.4234	12	1.449034	2.520918	329.5685
24	19	4.9403	1.81954	127.992	19	4.83572	1.83746	93.99454	18	4.961092	1.698598	60.67263
25	3	4.4068	1.26713	153.072	3	4.271902	1.243986	112.5013	3	4.164392	1.235821	73.42826
26	6	2.50719	1.73093	371.108	6	2.50701	1.789861	263.9193	7	2.506315	1.83375	167.1644
27	37	1.12904	4.98654	85.5081	49	1.235262	5	93.18909	15	1.122372	3.240711	62.1036
28	2	3.84029	1	173.942	2	3.577448	1	119.0599	2	3.476292	1	72.8206
29	15	4.60579	2.68295	140.685	15	4.635812	2.708626	98.17242	12	4.606094	2.56927	61.04882
30	19	4.7345	2.26292	119.119	18	4.723484	2.320777	86.85126	13	4.671073	2.428803	56.02209
31	49	1.23458	5	90.7123	49	1.234862	5	87.07998	19	1.089714	3.770908	57.9028
32	6	1.02531	2.32079	226.603	49	1.702005	1.693792	591.8705	6	1.046003	2.32274	202.4554
33	8	4.93353	1.19844	157.599	7	4.918013	1.118793	112.7648	6	4.857266	1	71.47633
34	22	4.83426	1.82485	148.779	21	4.831352	1.895135	104.9108	19	4.816409	1.831049	65.86319
35	6	4.46577	1.91817	299.636	6	4.430448	1.911524	238.8789	6	4.39221	1.904965	169.884
36	24	4.83772	1.49777	124.386	23	4.88388	1.3505	90.62342	23	4.861543	1.363861	58.97766
37	21	4.91675	2.3929	136.142	18	4.979047	1.867419	94.74836	21	4.917038	2.370524	59.47491
38	14	5.04971	2.86406	158.044	11	5.065401	2.551914	112.6085	7	4.351078	1.267174	70.69923
39	12	1	2.20222	1296.3	12	1	2.196867	1243.86	11	1	2.214968	1152.948
40	17	2.25976	2.96135	286.358	17	2.252952	2.982884	191.5385	18	2.247519	2.9979	115.1855
41	8	4.55457	2.28683	153.244	7	4.564318	2.127461	106.9183	3	4.105652	1	65.28106
42	49	1.23564	5	164.118	49	1.235393	5	157.3829	14	1.384042	2.100313	112.16
43	7	4.77231	1.25906	187.977	5	4.763225	1.200479	128.0479	3	4.743489	1	75.38152
44	17	1.99966	3.25761	196.216	18	1.994314	3.250432	137.0303	18	1.989812	3.244882	85.44086

Table 9. (continued)

	Standard z_{2i}				Two times z_{2i}				Four times z_{2i}			
Run	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$
45	50	1.31109	4.48334	115.586	50	1.445349	3.548975	116.4024	50	1.416924	3.724608	105.5867
46	10	6.75992	1	133.78	11	6.763387	1.005959	103.4579	6	6.602064	1	64.85695
47	33	1.67409	5	570.623	33	1.672756	5	402.1666	33	1.671583	5	252.7876
48	49	1.24214	4.79883	1115.2	49	1.242186	4.79775	1070.809	7	1.130644	2.242838	355.8343
49	2	4.6476	1	137.464	7	4.83887	1	114.8462	9	4.856132	1	78.35795
50	17	2.27403	2.89317	236.29	17	2.267617	2.878594	166.9051	17	2.262124	2.867554	105.1161
51	21	4.9021	2.50352	117.57	18	4.934234	1.945037	85.64356	16	4.925178	1.846194	55.56261
52	29	1.88527	2.94159	88.9871	29	1.902192	2.854945	85.43983	29	1.909508	2.777987	79.21479
53	3	3.73431	1	219.918	3	3.668461	1	150.1345	3	3.622036	1	91.72953
54	17	2.30593	3.16183	219.259	17	2.292183	3.144546	146.7096	17	2.281794	3.132989	88.2461
55	16	1.69484	2.46356	169.738	14	1.609242	2.482707	161.1894	30	1.943139	2.596918	201.9176
56	11	1.30226	2.55829	280.064	11	1.302823	2.558131	268.6971	11	1.381419	2.537217	249.269
57	2	4.65149	1	150.111	7	4.844281	1	120.6591	6	4.805428	1.060416	72.9769
58	12	1.47347	2.51426	273.706	13	1.501632	2.507532	264.3518	11	1.29597	2.559692	239.543
59	50	1.64956	2.43712	98.3792	50	1.649556	2.437125	94.41848	50	1.649549	2.437126	87.37731
60	12	4.55665	2.92537	127.551	12	4.558096	2.824466	91.86463	9	4.591579	2.384157	58.70866
61	28	1.67509	3.54884	128.689	28	1.624544	3.616837	123.1558	21	1.648104	3.018126	110.8188
62	14	4.89502	1.49059	165.582	12	4.950367	1.369452	113.721	11	4.956132	1.317773	70.49338
63	29	1.8613	3.04018	55.0029	29	1.846315	3.097408	52.66987	29	1.862127	3.037331	48.72995
64	3	3.36542	1	208.412	3	3.332655	1	148.8365	3	3.3053	1	94.63649
65	7	1.12835	2.24429	340.112	8	1.141319	2.235113	327.7509	8	1.180521	2.206038	307.6475
66	4	2.82212	1.4069	230.987	4	2.807059	1.421018	164.0833	4	2.79405	1.433271	103.8566
67	6	5.94585	1	100.274	20	6.425616	1	166.8402	20	6.457348	1	132.7649
68	7	4.92339	1.20122	172.767	6	4.903263	1.113538	120.7256	5	4.904113	1	75.07202
69	12	2.15721	2.43705	198.868	12	2.147454	2.496591	133.047	12	2.139014	2.532349	80.02037
70	2	4.13535	1	144.953	2	4.027657	1	106.6892	2	3.954081	1	69.76495
71	8	4.90599	1.2635	171.111	2	4.674731	1	101.2045	5	4.876127	1.053185	68.18065
72	2	2.67896	1	185.09	2	2.73238	1	132.5519	2	2.767003	1	84.47028
73	34	1	5	734.137	34	1	5	704.0547	34	1	5	650.6865
74	12	4.55665	2.92537	127.551	12	4.558096	2.824466	91.86463	9	4.591579	2.384157	58.70866
75	7	6.61145	1.78959	132.479	7	6.836096	1.642363	107.2128	6	6.567586	1.790004	78.52991
76	6	5	3	132.463	6	5	3	100.0912	6	5	3	67.21365
77	9	1	2.25231	745.989	10	1.232205	2.216836	713.9839	9	1	2.254229	662.013
78	7	4.78219	1.22387	170.418	5	4.742269	1.153651	114.1748	5	4.781296	1.040701	70.49271
79	8	1.12311	2.34099	238.287	7	1.005899	2.418405	213.6493	28	1.645126	1.949441	463.0769
80	29	1.82902	3.16842	39.5404	29	1.853656	3.082905	38.03673	29	1.834471	3.145026	35.04951
81	8	4.06379	1.91795	148.766	8	3.720095	1.673329	107.0688	9	3.566356	1.676286	68.35201
82	5	6.35491	1.47663	119.654	5	6.343455	1.473544	100.1918	5	6.331992	1.4699	75.52036
83	3	3.47689	1	173.252	3	3.440005	1	124.096	3	3.410833	1	79.11953
84	50	1.64956	2.43712	98.3792	50	1.649556	2.437125	94.41848	50	1.649549	2.437126	87.37731
85	14	6.32552	2.97878	97.3265	14	6.308128	2.981369	78.59408	14	6.310613	2.975681	56.72985
86	4	2.25438	1	431.264	4	2.292974	1	293.7297	4	2.318977	1	179.0985
87	4	5.6451	2.00752	146.698	2	6.371541	1	97.71232	2	6.349582	1	68.21924
88	4	2.82212	1.4060	230.987	4	2.807059	1.421018	164.0833	4	2.79405	1.433271	103.8566

6.5 The Impact of Changes in Weibull Distribution Parameters

6.5.1 Changes in Weibull Distribution Shape Parameter

By increasing the value of Weibull shape parameter, the values of optimal parameters of n and L and $E(A)$ decrease and the values of optimal parameters of h_1 increase.

6.5.2 Changes in Weibull Distribution Scale Parameter

In this section, by considering three different prior distributions, λ_i changes and its impact on the optimal values of loss cost will be discussed. In this paper, the negative exponential distribution, uniform and half-normal distributions are considered as a prior distribution. In Table 3, one can compare the optimal value of loss cost based on three prior distributions. As seen in this table, optimal values of loss cost based on three prior distributions have little difference with each other.

In Table 10, the results of how the optimal values of loss cost are affected by increasing values of Weibull distribution scale parameter is presented. The numerical results show that if the amounts of are doubled or quadrupled, in general, the optimum values of and and will not change significantly but the optimum value of parameter. The results illustrated in Figure 5.

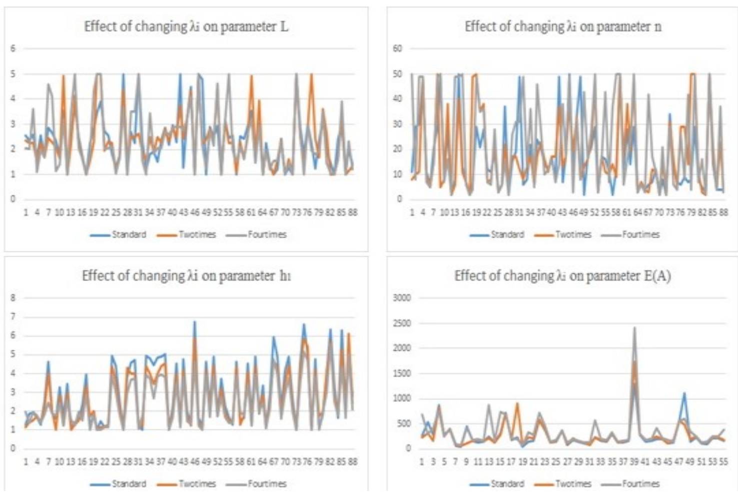

Figure 5. The relationship between parameter λ_i and design parameters and loss cost.

Table 10. Changes in optimal design parameters with variations in λ_i

	Standard λ_i				Two times λ_i				Four times λ_i			
Run	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$
1	11	1.30215	2.55756	270.695	8	1.169551	2.344766	237.6848	50	1.954323	2.063264	683.3481
2	29	1.89275	2.37332	536.061	10	1.41132	2.245094	307.2605	8	1.400085	2.00129	305.3701
3	30	1.94339	2.5964	281.897	11	1.496092	2.26051	165.7645	49	1.955669	3.619324	415.154
4	48	1.67787	1.68414	880.07	49	1.741973	1.50273	829.5358	49	1.80374	1.106479	749.4699
5	11	1.30226	2.55829	280.064	10	1.445557	2.264705	256.5509	7	1.363882	2.00095	242.4283
6	6	2.49973	1.75023	386.81	5	2.147675	1.713099	394.1555	5	1.845732	1.6732	399.9186
7	21	4.63033	2.88168	59.3397	15	4.018223	2.44378	74.85306	15	2.453407	4.589034	98.7464
8	29	1.93292	2.67074	38.6351	50	1.996466	2.297429	67.54971	49	1.89462	4.115418	75.25067
9	50	1.62729	2.37072	456.866	5	1	2.098363	117.3319	50	1.770976	1.150648	409.6016
10	9	3.27327	1.82437	157.23	8	2.807883	1.644324	165.2611	7	2.405567	1.408856	172.7665
11	28	1.67509	3.54884	128.689	38	1.243702	4.920375	174.3725	16	1.260044	3.220492	196.5423
12	2	3.47358	1	148.714	2	2.974643	1	156.2974	2	2.544676	1	163.4407
13	8	1.09431	2.43863	241.582	6	1	2.230296	204.8674	49	1.565955	3.513194	875.9763
14	50	1.41605	3.77037	128.391	40	1.287358	4.167116	131.7536	49	1.228852	5	157.2083
15	13	1.52744	2.50126	322.657	11	1.50474	2.246511	281.812	50	1.951425	2.046567	734.2502
16	7	2.35738	1.71054	715.689	7	1.895405	1.656005	708.9925	6	1.517505	1.604834	694.4706
17	2	3.94811	1	183.894	2	3.377801	1	194.8479	2	2.890348	1	205.5377
18	7	1.12897	2.24495	231.928	49	1.720186	1.536345	907.0007	4	1.002102	1.849993	192.0282
19	29	1.90034	2.83148	38.913	50	1.996457	2.297454	80.48257	49	1.843401	4.344352	77.61154
20	21	1	3.42575	151.747	35	1	5	227.3655	35	1	5	329.9785
21	28	1.47442	3.92026	162.946	38	1.1604	5	220.7433	36	1	5	279.7464
22	12	1.16974	2.71639	576.474	7	1.196265	1.951677	583.3938	7	1.170592	2.032434	725.993
23	11	1.29898	2.55879	368.039	8	1.233012	2.324557	385.8247	6	1.171481	2.064841	445.4872
24	19	4.9403	1.81954	127.992	23	4.407314	2.260242	135.831	28	3.923121	2.106442	144.5715
25	3	4.4068	1.26713	153.072	3	3.639091	1.166783	167.1227	3	3.0068	1.043627	180.514
26	6	2.50719	1.73093	371.108	6	2.155459	1.687613	371.9199	6	1.853632	1.641532	371.2991
27	37	1.12904	4.98654	85.5081	22	1.017281	4.39182	95.53965	15	1	3.678186	121.1338
28	2	3.84029	1	173.942	7	4.304809	1.595274	206.0268	2	2.979084	1	196.8829
29	15	4.60579	2.68295	140.685	17	3.996487	2.498243	152.4754	26	3.70408	3.517767	161.2512
30	19	4.7345	2.26292	119.119	17	3.996544	2.498272	127.0053	31	3.725319	3.514773	134.6182
31	49	1.23458	5	90.7123	12	1.196237	2.617205	80.45797	31	1.048848	5	120.6777
32	6	1.02531	2.32079	226.603	8	1.246556	1.9136	224.9928	49	1.751082	1.531273	567.4582
33	8	4.93353	1.19844	157.599	13	4.389385	1.530496	185.6689	13	3.964983	1	194.5054
34	22	4.83426	1.82485	148.779	17	4.008093	2.504047	159.6928	36	3.736297	3.451008	172.1395
35	6	4.46577	1.91817	299.636	6	3.467299	1.869719	318.7974	5	2.693558	1.814543	333.4401
36	24	4.83772	1.49777	124.386	17	3.996464	2.498232	133.6618	46	3.853617	2.065322	143.5072
37	21	4.91675	2.3929	136.142	23	4.41132	2.255214	145.4271	28	3.943873	2.036652	155.5667
38	14	5.04971	2.86406	158.044	16	4.536213	2.825243	172.0159	10	3.814351	2.741318	184.4923
39	12	1	2.20222	1296.3	11	1	2.31691	1739.89	12	1	2.59778	2411.717
40	17	2.25976	2.96135	286.358	17	1.94389	2.895596	295.8818	16	1.672064	2.83	304.3424
41	8	4.55457	2.28683	153.244	17	3.996685	2.498343	176.344	7	3.520766	2.938511	174.0563
42	49	1.23564	5	164.118	37	1.328111	3.731233	190.1827	14	1.172324	2.905707	192.8318
43	7	4.77231	1.25906	187.977	13	4.167863	2.426201	246.4638	28	3.824327	2.776537	412.6599
44	17	1.99966	3.25761	196.216	17	1.720137	3.234619	202.6433	17	1.479748	3.210994	208.3893

Table 10. (continued)

	Standard λ_i				Two times λ_i				Four times λ_i			
Run	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$	n	h_1	L	$E(A)$
45	50	1.31109	4.48334	115.586	37	1.236496	4.317304	119.6317	49	1.357481	4.114659	170.8082
46	10	6.75992	1	133.78	19	5.883671	1.113028	174.2922	3	4.642861	1	146.4589
47	33	1.67409	5	570.623	33	1.440725	5	560.5141	32	1.239867	5	548.5121
48	49	1.24214	4.79883	1115.2	8	1.076308	2.198225	481.265	8	1	2.150746	594.0127
49	2	4.6476	1	137.464	13	4.182429	2.395649	203.303	43	3.903022	1.223198	340.5079
50	17	2.27403	2.89317	236.29	16	1.955981	2.830596	241.3665	16	1.682333	2.768768	245.5565
51	21	4.9021	2.50352	117.57	23	4.404159	2.3148	124.6794	28	3.926145	2.143754	132.9303
52	29	1.88527	2.94159	88.9871	47	1.752369	4.535269	120.6014	50	1.762088	4.623786	152.4846
53	3	3.73431	1	219.918	3	3.188803	1	231.6516	3	2.720778	1	242.9377
54	17	2.30593	3.16183	219.259	17	1.985492	3.128139	229.8177	16	1.709491	3.093412	239.6824
55	16	1.69484	2.46356	169.738	11	1.531069	2.241506	179.9814	43	1.376035	5	382.4399
56	11	1.30226	2.55829	280.064	10	1.445557	2.264705	256.5509	7	1.363882	2.00095	242.4283
57	2	4.65149	1	150.111	14	4.358019	1.08026	217.9168	37	3.906524	1.513578	355.4361
58	12	1.47347	2.51426	273.706	9	1.30935	2.30111	235.9467	50	1.951755	2.10998	770.583
59	50	1.64956	2.43712	98.3792	50	1.76512	1.631539	101.1679	50	1.766414	1.617741	111.4556
60	12	4.55665	2.92537	127.551	16	4.07284	2.53642	137.2554	6	3.478783	2.659582	142.3792
61	28	1.67509	3.54884	128.689	38	1.243702	4.920375	174.3725	16	1.260044	3.220492	196.5423
62	14	4.89502	1.49059	165.582	19	4.425406	1.681384	182.9713	24	3.971214	1.50456	201.2301
63	29	1.8613	3.04018	55.0029	48	1.912698	3.957627	74.75582	50	1.955811	2.946067	111.172
64	3	3.36542	1	208.412	3	2.881366	1	215.0401	3	2.464944	1	221.0102
65	7	1.12835	2.24429	340.112	7	1.180223	1.965237	374.3197	6	1.147166	1.991122	441.8055
66	4	2.82212	1.4069	230.987	4	2.436576	1.325731	235.7882	3	2.109074	1.219979	239.6138
67	6	5.94585	1	100.274	3	4.666154	1.002993	97.74966	42	4.78721	1.557402	338.8541
68	7	4.92339	1.20122	172.767	12	4.381951	1.58014	205.6637	17	3.954685	1.531663	239.0508
69	12	2.15721	2.43705	198.868	11	1.854504	2.373075	209.4024	11	1.593932	2.303249	219.3426
70	2	4.13535	1	144.953	2	3.471622	1	154.5953	2	2.910318	1	163.7444
71	8	4.90599	1.2635	171.111	15	4.432757	1.618445	216.25	21	3.777733	1.489941	263.261
72	2	2.67896	1	185.09	2	2.281711	1	191.1031	2	1.955357	1	196.5511
73	34	1	5	734.137	31	1	5	1186.371	29	1	5	1863.928
74	12	4.55665	2.92537	127.551	16	4.07284	2.53642	137.2554	6	3.478783	2.659582	142.3792
75	7	6.61145	1.78959	132.479	5	5.848426	1.371328	138.5341	4	5.158408	1.003072	145.3803
76	6	5	3	132.463	29	5.457458	2.876427	143.5633	26	4.668699	2.968483	158.744
77	9	1	2.25231	745.989	29	1.019818	5	1318.914	8	1	1.954237	1172.319
78	7	4.78219	1.22387	170.418	14	4.1683	2.43118	226.12	42	3.844762	1.816726	403.1582
79	8	1.12311	2.34099	238.287	50	1.758336	1.667466	891.0237	4	1	1.928368	179.7044
80	29	1.82902	3.16842	39.5404	50	1.961812	3.595236	56.95504	50	1.967997	3.496289	74.06845
81	8	4.06379	1.91795	148.766	11	4.102607	2.633407	157.2931	7	3.020235	1.445944	165.2033
82	5	6.35491	1.47663	119.654	3	5.69928	1	123.8773	16	5.593009	1.061762	186.4104
83	3	3.47689	1	173.252	2	2.973015	1	179.8384	2	2.539934	1	185.88
84	50	1.64956	2.43712	98.3792	50	1.76512	1.631539	101.1679	50	1.766414	1.617741	111.4556
85	14	6.32552	2.97878	97.3265	17	5.328488	3.309648	113.4397	22	4.376038	3.916014	130.0003
86	4	2.25438	1	431.264	4	1.930286	1	435.3631	4	1.652991	1	437.9437
87	4	5.6451	2.00752	146.698	23	6.103531	1.141564	239.0092	37	5.058227	2.313293	352.1961
88	4	2.82212	1.4060	230.987	4	2.436576	1.325731	235.7882	3	2.109074	1.219979	239.6138

7 Conclusions Remarks and Future Researches

In this study, an approach was proposed for the design of a control chart having economic and statistical properties for a process in which the failure mechanism follows a Weibull shock model. This investigation mainly combined our cost model with Yang and Hancock's correlation model to develop an economic statistical design model of charts for processes with correlated data. In addition, modified Taguchi's loss function was incorporated into our economic statistical design of $\overline{X}$ control charts by redefining the in-control and the out-of-control costs.

The resulting model combines the advantages of the economic statistical design and Taguchi's philosophy. In practice, multiple assignable causes are more realistic than the single ones. To provide more protection for both consumers and producers, an economic statistical model of $\overline{X}$ control chart integrated with Taguchi loss function for correlated data under Weibull shock model with multiple assignable causes was provided.

The cost model under multiple assignable causes and single assignable cause compared with each other. In addition, the economic statistical design with non-uniform and uniform sampling schemes are compared. Based on above comparison the cost model with non-uniform sampling cost has a lower cost than that with uniform sampling.

By using sensitivity analysis, the effect of changing fixed and variable parameters of time, cost, correlation coefficient, Taguchi loss function parameters and Weibull distribution parameters on the optimum values of design parameters and loss cost examined and discussed.

For future research one can propose economic statistical model of T^2 control chart integrated with multivariate Taguchi loss function for correlated data under Weibull shock model with multiple assignable causes. The economic statistical design of control chart by considering variable design parameters under Weibull shock model for correlated data with multiple assignable causes is open problem for future research.

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Appendix A

Notations and Definitions

Z_0: Average time to search for false alarm.

Z_1: Average time to discover assignable cause once it is detected by control chart.

Z_{2i}: Average time to repair ith assignable cause after it has been discovered.

D_0: Average cost per unit of time while the process in control.

D_{1i}: Average cost per unit of time while the process is out of control owing to the occurrence of the ith assignable cause $(i = 1, 2, \dots, s)$.

L_{in}: Approximated in control cost obtained by considering modified Taguchi loss function.

$L_{out,i}$: Approximated out of control cost obtained by considering modified Taguchi loss function.

Y: The average cost per false alarm when the process is under control.

W_i: Cost to locate and repair ith assignable cause.

a: Fixed sample cost.

b: Unit sample cost.

P: Production rate.

Δ: Tolerance rate.

A: The cost to society for manufacturing a product out of specification.

σ: Standard deviation of the process.

λ_i: Weibull Scale parameter.

k: Weibull Shape parameter.

n: Sample size.

h_1: Sampling interval.

L: Control limits coefficient.

ρ: Average correlation factor within samples.

Appendix B

Regarding of h_j we have:

$$h_j = [j^{\frac{1}{k}} - (j-1)^{\frac{1}{k}}]h_1.$$

We show h_j is decreasing based of j:

$$\begin{aligned}\frac{\partial}{\partial j}h_j &= \left[\frac{1}{k}j^{\frac{1}{k}-1} - \frac{1}{k}(j-1)^{\frac{1}{k}-1}\right]h_1\\ &= [j^{\frac{1}{k}-1} - (j-1)^{\frac{1}{k}-1}]\frac{h_1}{k}\end{aligned}$$

h_1 is sampling interval and positive. In the above formula, $\frac{1}{k}$ is positive, because the shape parameter of Weibull distribution $K \geqslant 1$. Hence $j^{\frac{1}{k}-1} - (j-1)^{\frac{1}{k}-1} \leqslant 0$, therefore we have: $\frac{\partial}{\partial j}h_j \leqslant 0$. Because of above reason, h_j is decreasing based of j.

Appendix C

Recall that the $X \sim N(\mu, V)$, with $V = \sigma^2 R$ is the process variance and R

is the correlation matrix.

$$
\begin{aligned}
Var(\overline{X}) &= \frac{1}{n^2}\left[\sum_{i=1}^{n} Var(X_i) + \sum_{i\neq j} Cov(X_i, X_j)\right] \\
&= \frac{1}{n^2}\left[n\sigma^2 + \sum_{i\neq j} V_{ij}\right] \\
&= \frac{1}{n^2}\left[n\sigma^2 + \sigma^2\left\{\sum_{i\neq j} r_{ij}\right\}\right].
\end{aligned}
$$

Let $\rho = \frac{\sum_{i\neq j} r_{ij}}{n(n-1)}$. Then

$$
Var(\overline{X}) = \frac{1}{n^2}[n\sigma^2 + \sigma^2 n(n-1)\rho]
$$

$$
= \frac{\sigma^2}{n}[1 + (n-1)\rho].
$$

Appendix D

Regarding equation (6) ω_j and h_j can be obtained as follows:

$$
\int_{\omega_j}^{\omega_{j+1}} r_i(t)\,\mathrm{d}t = \int_{\omega_0}^{h_1} r_i(t)\mathrm{d}t, \qquad j = 1, 2, \ldots
$$

$$
\begin{aligned}
\int_{\omega_j}^{\omega_{j+1}} \lambda_i k t^{k-1}(t)\,\mathrm{d}t &= \int_{\omega_0}^{h_1} \lambda_i k t^{k-1}(t)\,\mathrm{d}t \\
&= \omega_{j+1}^k - \omega_j^k = h_1^k \\
\text{If} \quad j = 1 : \omega_2^k &= \omega_1^k + h_1^k \Rightarrow \omega_2 = 2^{\frac{1}{k}} h_1 \\
\text{If} \quad j = 2 : \omega_3^k &= \omega_2^k + h_1^k \Rightarrow \omega_3 = 3^{\frac{1}{k}} h_1 \\
&\vdots
\end{aligned}
$$

Therefore $\omega_j = j^{\frac{1}{k}} h_1$ and

$$\begin{aligned} h_j &= \omega_j - \omega_{j-1} \\ &= [j^{\frac{1}{k}} - (j-1)^{\frac{1}{k}}]h_1, \qquad j = 1, 2, \ldots. \end{aligned}$$

Appendix E

Let $AATS_i$ be the average time between occurrence shifts in process mean owing to ith assignable cause and receiving true alarm from control chart.

$$\begin{aligned} AATS_i &= \sum_{j=1}^{\infty}\left[\sum_{k=1}^{\infty} q_{ij}[\omega_{k+j-1} - \omega_{j-1}]\beta_i^{k-1}(1-\beta_i)\right] - \tau_i \\ &= \sum_{j=1}^{\infty}\left[\sum_{k=1}^{\infty}(1-p_i)^{j-1}p_i[\omega_{k+j-1} - \omega_{j-1}]\beta_i^{k-1}(1-\beta_i)\right] - \tau_i \\ &= (1-\beta_i)p_i \sum_{j=1}^{\infty}\left[\sum_{k=1}^{\infty}[\underbrace{(1-p_i)^{j-1}\omega_{k+j-1}\beta_i^{k-1}} - \underbrace{(1-p_i)^{j-1}\omega_{j-1}\beta_i^{k-1}}]\right] - \tau_i. \end{aligned}$$

For the first part, we have:

$$\begin{aligned} I &= \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}(1-p_i)^{j-1}\omega_{k+j-1}\beta_i^{k-1} \\ &= \sum_{l=1}^{\infty}\sum_{j=1}^{l}(1-p_i)^{j-1}\omega_l\beta_i^{l-j} \\ &= \sum_{l=1}^{\infty}\omega_l\beta_i^l\sum_{j=1}^{l}(1-p_i)^{-1}(\frac{1-p_i}{\beta_i})^j \\ &= (\frac{1}{1-p_i})\sum_{l=1}^{\infty}\omega_l\beta_i^l(\frac{1-p_i}{\beta_i-1+p_i}) - (\frac{1}{1-p_i})\sum_{l=1}^{\infty}\omega_l(\frac{(1-p_i)^{l+1}}{\beta_i-1+p_i}) \\ &= (\frac{1}{p_i+\beta_i-1})(\sum_{l=1}^{\infty}\omega_l\beta_i^l - \sum_{l=1}^{\infty}\omega_l(1-p_i)^l) \\ &= (\frac{h_1}{p_i+\beta_i-1})(\beta_i A(\beta_i) - (1-p_i)A(1-p_i)) \end{aligned}$$

For the second part, we have:

$$
\begin{aligned}
II &= \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}(1-p_i)^{j-1}\omega_{j-1}\beta_i^{k-1} \\
&= \sum_{j=1}^{\infty}(1-p_i)^{j-1}\omega_{j-1}\sum_{k=1}^{\infty}\beta_i^{k-1} \\
&= \frac{h_1(1-p_i)}{1-\beta_i}A(1-p_i).
\end{aligned}
$$

By substituting and simplifying, final formula is obtained.

Appendix F

If B_i is the event of the occurrence of ith assignable cause, then

$$P(B_i) = P(T_i < Y),$$

where $Y = \min(T_1, T_2, \ldots, T_{i-1}, T_{i+1}, \ldots, T_s)$

Let $\lambda^{'} = \sum_{i \neq j}$ and $\lambda_0 = \sum \lambda_j, j = 1, 2, \ldots, s$, then

$$
\begin{aligned}
P(B_i) &= P(T_i < Y) \\
&= \int_0^{+\infty} P(T_i < Y|Y = y)f_Y(y)\mathrm{d}y \\
&= \int_0^{+\infty} P(T_i < Y)f_Y(y)\mathrm{d}y \\
&= \int_0^{+\infty} (1-e^{-\lambda_i y^k})\lambda^{'} k y^{k-1} e^{-\lambda y^k}\mathrm{d}y \\
&= \int_0^{+\infty} \lambda^{'} k y^{k-1} e^{-\lambda y^k}\mathrm{d}y - \int_0^{+\infty} e^{-\lambda_i y^k}\lambda^{'} k y^{k-1} e^{-\lambda y^k}\mathrm{d}y \\
&= 1 - \lambda^{'}\int_0^{+\infty} k y^{k-1} e^{-y^k(\lambda_i+\lambda^{'})}\mathrm{d}y \\
&= 1 - \frac{\lambda^{'}}{\lambda_i+\lambda^{'}} = \frac{\lambda_i}{\lambda_i+\lambda^{'}} = \frac{\lambda_i}{\lambda_0}.
\end{aligned}
$$

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