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Admissible and Minimax Estimator of the Parameter θ in a Binomial Bin(n,θ)-distribution under Squared Log Error Loss Function in a Lower Bounded Parameter Space

Mohammad Jafari Jozani

Allameh Tabatabaie University

Extended Abstract. The study of truncated parameter space in general is of interest for the following reasons:

1. They often occur in practice. In many cases certain parameter values can be excluded from the parameter space. Nearly all problems in practice have a truncated parameter space and it is most impossible to argue in practice that a parameter is not bounded.

2. In truncated parameter space, the commonly used estimators of θ such as the maximum likelihood estimators are inadmissible. Even more characteristic is the fact that boundary rules are mostly inadmissible, where a boundary estimator is an estimator which takes, with positive probability for some $\theta \in \Theta$, values on or near the boundary of Θ.

3. In truncated parameter space, the frequently used criterion of unbiasedness is useless, since no unbiased estimator exists in general. So, other optimality criteria have to be looked for, such as admissibility, minimaxity and invariance among others (Moors, 1985).

So, estimation in truncated parameter space is one of the most important features in statistical inference. As mentioned above, most of the frequently used estimators such as the MLE are inadmissible in truncated parameter space. One way to avoid this difficulties is using other criteria of optimality, such as minimaxity. Minimaxity of estimators is not only of own interest, but it serves also as a useful benchmark to measure the performance of estimators.

It has played an important role in estimation problems in restricted parameter spaces since the work of Katz (1961) who considered minimax and admissible estimator of a normal mean in a lower bounded parameter space under squared error loss function.

Minimax estimation problems with restricted parameter space reached increasing interest within the last two decades. For a review of minimax estimation in truncated parameter space, see Bischoff (1992), Bader and Bischoff (2003), Jafari Jozani et al. (2002), Marchand and Parsian (2006), Marchand and Strawderman (2004), van Eeden (2006), and Jafari Jozani and Marchand (2007) among others.

In this paper, estimation of a lower bounded parameter $\theta \in [b, 1]$ in a binomial distribution, $\mathrm{Bin}(n, \theta)$, with probability function

$$P(X = x) = \binom{n}{x} \theta^x (1-\theta)^{n-x}, \qquad x = 0, \ldots, n$$

under the squared log error loss function

$$\mathrm{L}(\theta, \delta) = (\log \delta - \log \theta)^2 = \left\{ \log \left(\frac{\delta}{\theta} \right) \right\}^2$$

is considered and minimaxity of a boundary supported prior Bayes estimator of $\theta \in [b, 1]$ is proved. More specifically, we focused on explicit conditions for the minimax estimator to be Bayes with respect to a boundary supported prior. To this end, we identified the unique "equalizer" rule among two point boundary priors as well as its risk function. Also a necessary and sufficient condition for the equalizer rule to be minimax is presented.

Keywords. minimax estimation; restricted parameter space; squared log error loss; binomial distribution; lower bounded parameter space; two-point prior.

References

Bader, G.; Bischoff, W. (2003). Old and new aspects of minimax estimation of a bounded parameter. In *Mathematical Statistics and Applications: Festschrift for Constance van Eeden*, M. Moore, S. Froda, and C. Leger, eds. IMS Lecture Notes Monogr. Ser. **42**, Inst. Math. Stat., Beachwood, OH, pp. 15-30.

Bischoff, W. (1992). Minimax estimation and Γ-minimax estimation for functions of a scale parameter family under L_p loss. *Statist. Decisions* **29**, 45-61.

van Eeden, C. (2006). *Restricted Parameter Space Estimation Problems: Admissibility and Minimaxity Properties.* Springer, New York.

Jafari Jozani, M.; Marchand, E. (2007). Minimax estimation of constrained parametric functions for discrete families of distributions. To appear in *Metrika.*

Jafari Jozani, M.; Nematollahi, N.; Shafie, K. (2002). Admissible and minimax estimator of a bounded scale-parameter in a subclass of the exponential family under scale-invariant squared error loss. *Statist. Probab. Lett.* **60**, 437-444.

Marchand, E.; Parsian, A. (2006). Minimax estimation of a bounded discrete parameter. *Statist. Probab. Lett.* **76**, 547-554.

Marchand, E.; Strawderman, W.E. (2004). Estimation in restricted parameter space: A review. In *A Festschrift for Herman Rubin,* A. Dasgupta, ed. IMS Lecture Notes Monogr. Ser. **45**, Inst. Math. Stat., Beachwood, OH, pp. 21-44.

Moors, J.J.A. (1985). Estimation in truncated parameter spaces. Ph.D. thesis, Tilburg University, Tilburg, The Netherlands.

Mohammad Jafari Jozani
Department of Statistics,
Faculty of Economics,
Allameh Tabatabaie University,
Tehran, Iran.
e-mail: *m.j.jozani@srtc.ac.ir*

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