Journal of Statistical Research of Iran
Vol. 1 No. **2** pp. 121-122

مجلّه‌ی پژوهش‌های آماری ایران
دوره‌ی ۱، شماره‌ی ۲، پاییز و زمستان ۱۳۸۳، صص ۱۸۷-۱۸۸

Estimation of Scale Parameter in a Subfamily of Exponential Family with Weighted Balanced Loss Function

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Extended Abstract. Suppose $X_1, \ldots, X_n$ is a random sample of size n from a distribution with pdf $\frac{1}{\tau} f(\frac{x}{\tau})$, where f is known and τ is an unknown positive scale parameter. Then the joint density is denoted by

$$f_\tau(\mathbf{x}) = \Pi_{i=1}^n \frac{1}{\tau} f\left(\frac{x_i}{\tau}\right). \tag{1}$$

In many cases the model (1) reduces to (see Parsian amd Nematollahi, 1994)

$$f_\theta(\mathbf{x}) = c(\mathbf{x}, n)\theta^{-n\nu} \exp\left\{-\frac{T(\mathbf{x})}{\theta}\right\}, \tag{2}$$

where $c(\mathbf{x}, n)$ is a function of $\mathbf{x}$ and $n, \theta = \tau^r$ for some r, ν is a function of n and $T(\mathbf{X}) = \sum_{i=1}^n T(X_i)$ is a complete sufficient statistic for θ with $\Gamma(n\nu, \theta)$-distribution.

Examples of such models are: $\Gamma(\alpha, \beta)$ with α known and $\theta = \beta$; $E(0, \beta)$ with $\theta = \beta$; $N(0, \sigma^2)$ with $\theta = \sigma^2$.

This paper considers estimation of θ and studies admissibility and inadmissibility of the obtained estimators in a subfamily of exponential family under a Weighted Balanced Loss Function (WBLF).

The BLF is defined as

$$L_B(\hat{\theta}, \theta) = \frac{\omega}{n} \sum_{i=1}^n (T(X_i) - \hat{\theta})^2 + (1-\omega)(\hat{\theta} - \theta)^2, \tag{3}$$

where $0 \leqslant \omega \leqslant 1$ and $\hat{\theta}$ is an estimate of θ. The BLF, introduced by Zellner (1994), is formulated to reflect two criteria, namely goodness of fit and precision

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of estimation. Rodrigues and Zellner (1994) considered the estimation problem of the exponential mean time to failure under a WBLF. The WBLF may be defined as follows

$$L_{WB}(\hat{\theta},\theta)=\frac{\omega}{n}q(\theta)\sum_{i=1}^{n}(T(X_i)-\hat{\theta})^2+(1-\omega)q(\theta)(\hat{\theta}-\theta)^2 \qquad (4)$$

where $q(\theta)$ is a suitably chosen positive function of θ.

Under the BLF (3), Chung and Kim (1997) showed that the usual estimator $\overline{\mathbf{X}}$ is admissible when $p \leqslant 2$ and it is inadmissible for $p \geqslant 3$, where $\overline{\mathbf{X}} = (\overline{X}_1, \cdots, \overline{X}_p)'$. They also, showed that $\overline{\mathbf{X}}$ is minimax. Sanjari Farsipour and Asgharzadeh (2003) obtained the Bayes estimator of the mean vector θ and the covariance matrix Σ. They also discussed the admissibility of $c\overline{\mathbf{X}}+d$ for the mean vector. Under BLF (3) and WBLF (4), Chung *et. al.* (1998), Sanjari Farsipour and Asgharzadeh (2003, 2004) discussed the admissibility and inadmissibility of estimators of the form $a\overline{\mathbf{X}}+b$ when the probability model is Poisson and Normal respectively. Dey *et. al.* (1999) studied the notion of a BLF from the perspective of unifying a variety of results both frequentist and Bayesian. They showed in broad generality that frequentist and Bayesian results for BLF follow from and also imply related results for quadratic loss function reflecting only precision of estimation.

In this paper, we determine the suitable choice of $q(\theta)$ and derive the Bayes estimators of $\theta=\tau^r$ under the loss (4). We characterize the class of inadmissible and admissible linear estimators of the form $A\overline{T}+B$ for all possible values of A and B. Empirical Bayes estimator of θ under the loss (4) is obtained and classified as an admissible estimator. Bayes estimation of θ is considered using noninformative and informative hierarchical models and it is shown that the Bayes estimator of θ under the WBLF (4) using suitable noninformative hierachical model is minimax and admissible.

Keywords. admissibility; exponential family; weighted balanced loss function; Bayes estimator; empirical Bayes estimator; hierarchical Bayes estimator.

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The full version of the paper, in Persian, appears on pages 168-186.