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Distribution Free Confidence Intervals for Quantiles Based on Extreme Order Statistics in a Multi-Sampling Plan

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Extended Abstract. Let $X_{i1},\ldots,X_{in_i}$, $i=1,\ldots,k$, be independent random samples from distribution F^{α_i}, where F is an absolutely continuous distribution function and $\alpha_i>0$. Also, suppose that these samples are independent. Let M_{i,n_i} and M'_{i,n_i}, respectively, denote the maximum and minimum of the ith sample. Constructing the distribution-free confidence intervals for quantiles of F based on these informations is the aim of this paper. Various cases have been studied and in each case, the exact non-parametric confidence intervals are obtained. First, we concentrate our attention to the **maxima** of the samples. Coverage probability of a confidence interval based on two different maxima $(i,j;i\neq j)$, is derived as follows

$$P(M_{i,n_i}\leqslant \xi_p\leqslant M_{j,n_j})=p^{\alpha_i n_i}(1-p^{\alpha_j n_j}).$$

For the case of $P(M_{i,n_i}>M_{j,n_j})=1$, two schemes are suggested as:

1. Using the ordered statistics of M_{i,n_i} and M_{j,n_j} as $M_{ij1}\leqslant M_{ij2}$. In this case the associated confidence coefficient is derived as follows

$$P(M_{ij1}\leqslant \xi_p\leqslant M_{ij2})=p^{\alpha_i n_i}(1-p^{\alpha_j n_j})+p^{\alpha_j n_j}(1-p^{\alpha_i n_i}).$$

2. Utilizing the ordered maxima of the k samples as $M_{1:k}\leqslant M_{2:k}\leqslant\ldots\leqslant M_{k:k}$. Using $(M_{i:k},M_{j:k})$, $i<j$, as a confidence interval for ξ_p, confidence coefficient is obtained, as

$$P(M_{i:k}\leqslant \xi_p\leqslant M_{j:k})=\sum_{r=i}^{j-1}\sum_{A_r}p^{\sum_{s=1}^{r}n_{t_s}\alpha_{t_s}}\prod_{s=r+1}^{k}(1-p^{n_{t_s}\alpha_{t_s}}),$$

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where the summation on A_r extends over all permutations $(t_1, ..., t_k)$ of $\{1, ..., k\}$ for which $t_1 < \cdots < t_r$ and $t_{r+1} < \cdots < t_k$. Several special cases are considered, say, for $\alpha_i = 1$ and $n_i = n$, $i = 1, ..., k$, the last identity is simplified as follows

$$P(M_{i:k} \leqslant \xi_p \leqslant M_{j:k}) = \sum_{r=i}^{j-1} \binom{k}{r} p^{nr}(1-p^n)^{k-r},$$

that introduce a binomial representation. By this property, it can be identified i and j to achieve the desired qualities. For other special cases such as $\alpha_i = \frac{1}{n_i}$ or $\alpha_i = \frac{n^*}{n_i}$, $i = 1, ..., k$, where $n^* = \sum_{i=1}^k n_i$, the similar relations are obtained. Similar results for **minima** are obtained. Specially, the following relations are derived

$$P(M'_{ij1} \leqslant \xi_p \leqslant M'_{ij2}) = \{1-(1-p^{\alpha_i})^{n_i}\}(1-p^{\alpha_j})^{n_j} + \{1-(1-p^{\alpha_j})^{n_j}\}(1-p^{\alpha_i})^{n_i},$$

where $M'_{ij1} < M'_{ij2}$ denote the ordered minima of two different samples. Also,

$$P(M'_{i:k} \leqslant \xi_p \leqslant M'_{j:k}) = \sum_{r=i}^{j-1} \sum_{A_r} \left\{1 - \left(1 - p^{\alpha_{t_s}}\right)^{n_{t_s}}\right\} \prod_{s=r+1}^{k} \left(1 - p^{\alpha_{t_s}}\right)^{n_{t_s}},$$

where $M'_{1:k} < M'_{2:k} < ... < M'_{k:k}$ are order statistics of M'_{i,n_i}'s, $i = 1, ..., k$.

Finally, both **minima** and **maxima** are used to construct the distribution-free confidence intervals for ξ_p in independent and dependent situations. It is clear that M'_{i,n_i} and M_{j,n_j}, $i \neq j$, are independent and their ordered statistics are denoted by $V_{ij1} < V_{ij2}$. Therefore, the following coverage probability is obtained as

$$P(V_{ij1} \leqslant \xi_p \leqslant V_{ij2}) = \{1 - (1 - p^{\alpha_i})^{n_i}\}(1 - p^{\alpha_j n_j}) + p^{\alpha_j n_j}(1 - p^{\alpha_i})^{n_i}.$$

In the other hand, by choosing (M'_{i,n_i}, M_{i,n_i}) as a confidence interval, we deal with a dependent end points interval that the confidence coefficient is derived as follows

$$P(M'_{i,n_i} \leqslant \xi_p \leqslant M_{i,n_i}) = 1 - p^{\alpha_i n_i} - (1 - p^{\alpha_i})^{n_i}.$$

Keywords. Extreme order statistics; confidence interval; quantile; coverage probability; F^α model; reversed hazard rate function.

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