© Statistical Research and Training Center
مجلّه‌ی پژوهش‌های آماری ایران
پژوهشکده‌ی آمار
J. Statist. Res. Iran **6** (2009): 161–163
دوره‌ی ۶، شماره‌ی ۱، بهار و تابستان ۱۳۸۸، صص ۱۶۱–۱۶۳

Empirical Likelihood Approach and its Application on Survival Analysis

Mohadese Safakish* and Hamidreza Navvabpour

Allameh Tabatabaie University

Extended Abstract. A number of nonparametric methods exist when studying the population and its parameters in the situation when the distribution is unknown. Some of them such as "resampling bootstrap method" are based on resampling from an initial sample.

In this article empirical likelihood approach is introduced as a nonparametric method for more efficient use of auxiliary information to construct confidence regions.

In empirical likelihood approach a Lagrange multipliers method is applied to estimate p_i, probability mass on ith observation (x_i), where $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1$, such that empirical likelihood function, $L(F) = \prod_{i=1}^{n} p_i = \prod_{i=1}^{n} dF(x_i)$, is maximized and an unbiased estimating equation, $\sum_{i=1}^{n} p_i g(x_i, \theta) = 0$ satisfied.

In order to use the method in different areas of statistics to compute confidence regions, it is sufficient to define an appropriate estimating equation. To extend the implementation of this method in survival analysis and construct confidence region for parameter vector of median regression model $m_i = \beta' Z_i + \varepsilon_i$, where Z_i is a vector of auxiliary variables, β a vector of parameters, and m_i a median life-length, Qin and Tsao (2003) consider the following estimating equation

$$\sum_{i=1}^{n} p_i Z_i \left\{ \frac{I(Y_i \geqslant \beta_0' Z_i)}{1 - \hat{G}(\beta_0' Z_i)} - \frac{1}{2} \right\} = \sum_{i=1}^{n} p_i W_{ni} \approx 0,$$

where $1 - \hat{G}(\cdot)$ is the Kaplan-Meier estimator of survival function of censored variable. Subramanian (2007) illustrates the concept of profile empir-

* Corresponding author

ical likelihood to make inference about a subset of parameters. Although the limiting distribution of log-empirical likelihood ratio function, $\ell(\beta_0) = 2\sum_{i=1}^{n}\log\{1+\lambda' W_{ni}\}$, is shown as weighted summation of independent chi-square variables, in this paper empirical likelihood bootstrap method was applied to build sampling distribution of the function. This approach uses a simple random sampling with unequal probability method to resample from an initial sample using empirical likelihood probabilities (p_i)'s. Also, there is an adjustment to avoid producing negative probabilities.

As an application of the above methods, Bone Marrow Transplantation data set was used to estimate parameters of the median life-length regression model in the presence of right censored data. This data set contains two auxiliary variables, Karnofsky score and waiting time to transplant (in months), a failure time, time to death or relapse (in days), and a death/relapse indicator which have been measured on 43 patients.

To find point estimation of parameters of the median life-length regression model a grid search method was applied and confidence region of vector of parameters through simulating the distribution of log-empirical likelihood ratio function was constructed. Non-symmetric 95% confidence interval for each regression coefficient illustrates that every individual regression coefficient comes from a skew distribution. While waiting time to transplant has no significant effect on median life-length after operation, Karnofsky score has a direct influence on it. We also obtain a table containing prediction of median life-length correspond to each separate values of Karnofsky index.

Keywords. Empirical likelihood; estimating equation; Kaplan-Meier estimator; median regression; right censored; profile empirical likelihood; empirical likelihood bootstrap method.

References

Qin, G. and Tsao, M. (2003). Empirical likelihood inference for median regression models for censored survival data, *Journal of Multivariate Analysis*, **85**, 416-430.

Subramanian, S. (2007). Censored median regression and profile empirical likelihood, *Statistical Methodology*, **4**, 493-503.

Mohadese Safakish
Department of Statistics,
Faculty of Economic,
Allameh Tabatabaie University,
Tehran, Iran.
e-mail: *m.safakish_atu@yahoo.com*

Hamidreza Navvabpour
Department of Statistics,
Faculty of Economic,
Allameh Tabatabaie University,
Tehran, Iran.
e-mail: *h.navvabpour@srtc.ac.ir*

The full version of the paper, in Persian, appears on pages 121–139.