Partial Association Components in Multi-way Contingency Tables and Their Statistical Analysis

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Extended Abstract. In analyses of contingency tables made up of categorical variables, the study of relationship between the variables is usually the major objective. So far, many association measures and association models have been used to measure the association structure present in the table. Although the association measures merely determine the degree of strength of association between the study variables, the association models illustrate the details and components of association structure. These measures and models have found vast application in many disciplines. For more details see Goodman and Kruskal (1954, 1979), Leibetrau (1983), Goodman (1972, 1979, 1985, 1991) and Ghoreishi and Meshkani (2006, 2008).

When one is interested in demonstrating the association components of two ordinal categorical variables while there is at least one explanatory categorical variable, it is natural to think of partial association between the two ordinal variables while the effect of the explanatory variable(s) are averaged out in some sense. We believe that this approach is the best way to incorporate both the extra information available in explanatory variable(s) into analysis and interpret the role and share of various polynomial trends.

In this paper, we first define the orthogonal polynomial partial association components between any pair of ordinal categorical variables in multi-way contingency tables. This is achieved by considering $RC(M_1, M_2)$ association model for the contingency table under study. Association and hence partial association coefficients for any pair of variables constituting the contingency

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table are generally regarded as unknown parameters. Thus, one needs to estimate them from the observed contingency table.

Here, without loss of generality, we consider the partial association between ordinal categorical variables X (with I levels) and Y (with J levels) while there is a sole explanatory variable Z (with L levels).

The association model for this table is given below:

$$\ln m_{ijl} = \lambda_{\cdot(l)} + \lambda_{i(l)}^{X} + \lambda_{j(l)}^{Y} + \sum_{k=1}^{I-1} \sum_{k'=1}^{J-1} \phi_{kk'}^{(l)} \mu_{ik} \nu_{jk'},$$

with constraints

$$\sum_{i} w_i \mu_{ik} = 0, \quad \sum_{i} w_i \mu_{ik}^2 = 1, \quad \sum_{j} w'_j \nu_{jk'} = 0, \quad \sum_{j} w'_j \nu_{jk'}^2 = 1,$$

$$\sum_{i} w_i \mu_{ik} \mu_{ik'} = 0, \quad \sum_{j} w'_j \nu_{jk} \nu_{jk'} = 0, \quad \sum_{i} w_i = 1, \quad \sum_{j} w'_j = 1.$$

From model (1) the $k \times k'$ intrinsic association component between X and Y at $Z = l$ is defined as

$$\phi_{kk'}^{(l)} = \sum_{i=1}^{I} \sum_{j=1}^{J} w_i w'_j \mu_{ik} \nu_{jk'} \ln m_{ijl}.$$

Using (2), the $k \times k'$ partial intrinsic association component of (X, Y) for various levels of Z is

$$\psi_{kk'} = \sum_{i=1}^{L} \pi_{..l} \phi_{kk'}^{(l)}.$$

These partial intrinsic association components lead us to define the share of $k \times k'$ partial association components as

$$\kappa_{kk'} = \frac{\psi_{kk'}}{\sqrt{\sum_{u=1}^{I-1} \sum_{u'=1}^{J-1} (\psi_{uu'})^2}}, \quad k = 1, \ldots, I-1; \ k' = 1, \ldots, J-1.$$

It should be noted that this definition of partial association is in fact much different from our previous definition provided in Ghoreishi and Meshkani (2009). Here, we average out the effect of the explanatory variable Z on intrinsic association parameters whereas in our paper (2009), it is dealt with in a different sense.

Both Bayesian and frequentist approach to inference on the association

and partial association coefficients are considered. Bayesian and maximum likelihood estimators are derived, correspondingly. Their asymptotic distributions are obtained, as well. Also, it is shown that the proposed results apply to any non-polynomial, but orthogonal, components. Moreover, as an illustration of our results, two three-way tables containing real data sets are analyzed.

Keywords. association measures; association models; multiple and partial association; orthogonal polynomial components.

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